Institution: University of Surrey

Unit of Assessment: 10 Mathematical Sciences

Title of case study: Advancing the Development, Public Understanding and Benefits of Global Weather Forecasting

Period when the underpinning research was undertaken: 2004-2014

Details of staff conducting the underpinning research from the submitting unit:

Name(s):	Role(s) (e.g. job title):	Period(s) employed by submitting HEI:
Ian Roulstone	Professor of Mathematics	January 2004 – current
Andrew Lorenc	Visiting Professor	February 2010 – current

Period when the claimed impact occurred: 2014 - 2020

Is this case study continued from a case study submitted in 2014? N

1. Summary of the impact (indicative maximum 100 words)

To increase the benefits to society of weather forecasting, meteorological agencies need to develop more accurate prediction algorithms and invest heavily in computing infrastructure. University of Surrey research on data assimilation algorithms and the nonlinearity and geometry of models underpinning weather forecasting has guided the development of improved algorithms at the UK Met Office and Météo-France. The research has also informed and inspired a book and television documentary series promoting public understanding of weather forecasting.

2. Underpinning research (indicative maximum 500 words)

A series of meetings organised by Professor Roulstone at the Isaac Newton Institute in 1996, accompanied by two volumes of proceedings published in 2002, launched a world-leading programme on Data Assimilation (DA) and Numerical Weather Prediction (NWP) focusing on incorporating qualitative knowledge about atmospheric behaviour into quantitative models and algorithms, and on the roles of nonlinearity and geometry in this process. This section describes key contributions made by Roulstone and colleagues to this programme since his appointment at the University of Surrey in 2004.

Data Assimilation for Weather Forecasting

Four-Dimensional (space + time) Variational DA (4DVar) has been central to NWP for two decades. Forecasts are made using parameters obtained by minimising cost functions with two terms representing observation errors and model-dependent 'background' errors. For the latter traditional 4DVar uses a linear climatological model to provide flow-dependence over the assimilation window while ensemble DA methods estimate it by integrating a set of model states with slightly different initial conditions. The ensemble approach is potentially more accurate, and avoids costly maintenance of linear climatological models, but suffers from significant sampling errors if the ensemble size is too small, and so is itself computationally expensive. In 4D Hybrid Ensemble Variational (4DEnVar) DA this is mitigated by replacing the linear climatological model in traditional 4DVar by an appropriate linear combination of ensemble trajectories, a process called 'localisation'.

Unfortunately, comparisons using near operational NWP systems demonstrated inferior performance of ensemble DA methods relative to 4DVar. To understand the reasons for this, research undertaken with Met Office Scientists **[R1]**, compared traditional 4DVar and 4DEnVar for a toy model. It showed that if a perfect model is used the 4DEnVar outperforms 4DVar if the

ensemble is large enough, but is worse in the presence of large sampling errors and model errors. This is because the localisation operator in 4DEnVar does not commute with the model, resulting in the background errors not following the flow. The difference in performance is considerably reduced if some stochastic noise is added to the 4DEnVar ensemble members at each step, a process known as 'additive inflation'.

Nonlinearity and Geometry in Forecasting

A fundamental challenge facing NWP is the development of low-cost, accurate simulators of atmospheric models. These simulators are integrated into ensemble and DA schemes to mitigate the 'butterfly effect' caused by strong nonlinearities.

Following a strategy successful in other contexts, Roulstone and colleagues have explored approaches preserving the geometry and conserved quantitates of fluid and atmospheric models. The conserved quantities include Potential Vorticity (PV), a central concept in weather forecasting. The study **[R3]** revealed the relationship between several atmospheric model approximations that preserve conserved quantities while further work **[R4]** used the conservation of PV to propose a new, more efficient splitting of model variables for DA schemes. More broadly, how the Hamiltonian structure that underpins many fluid and atmospheric models might be integrated into DA schemes was demonstrated **[R2]**. Further, it was shown that quaternionic geometry can be used in an efficient reformulation of fluid flow equations **[R5]** while the geometry of differential forms can be used to characterise coherent structures in fluids **[R6]**.

3. References to the research (indicative maximum of six references)

[R1] D. Fairbairn, S.R. Pring, **A.C. Lorenc**, **I. Roulstone**, (**2013**), A comparison of 4DVar with ensemble data assimilation methods, *Quarterly Journal of the Royal Meteorological Society,* **140**, 281-294. DOI: 10.1002/qj.2135

[R2] L.R. Watkinson, A.S. Lawless, N.K. Nichols, **I. Roulstone**, (**2005**), Variational data assimilation for Hamiltonian problems, *International Journal For Numerical Methods In Fluids,* **47,** 1361-1367. DOI: 10.1002/fld.844

[R3] A.A. White, B.J. Hoskins, **I. Roulstone**, A. Staniforth, **(2005)**, Consistent approximate models of the global atmosphere: shallow, deep, hydrostatic, quasi-hydrostatic and non-hydrostatic, *Quarterly Journal of the Royal Meteorological Society,* **131**, 2081-2107. DOI: 10.1256/qj.04.49

[R4] M. Wlasak, N.K. Nichols, **I. Roulstone**, **(2006)**, Use of potential vorticity for incremental data assimilation, *Quarterly Journal of the Royal Meteorological Society,* **132**, 2867–2886. DOI: 10.1256/qj.06.02

[R5] J. D. Gibbon, D. D. Holm, R. M. Kerr, **I. Roulstone**, **(2006)**, Quaternions and particle dynamics in the Euler fluid equations, *Nonlinearity,* **19**, 1969-1983. DOI: 10.1088/0951-7715/19/8/011

[R6] **I. Roulstone**, B. Banos, J.D. Gibbon, V.N. Roubtsov, **(2009),** A geometric interpretation of coherent structures in Navier–Stokes flows, *Proceedings of the Royal Society of London, Series A,* **465**, 2015–2021. DOI: 10.1098/rspa.2008.0483

4. Details of the impact (indicative maximum 750 words)

Research by Professor Roulstone and colleagues has led to the development of more accurate prediction algorithms for meteorological agencies and has promoted public understanding of the mathematics and science of weather forecasting.

More accurate, longer-term and more local weather forecasts are of considerable benefit to society through impacts on agriculture, transport, flood-control, and water and energy supplies. The

Surrey research both advances the ability of meteorological agencies to increase these benefits by directly increasing agencies' forecasting skills and by increasing public understanding of weather forecasting.

Developing accurate prediction algorithms

Surrey research has contributed significantly to guiding research and development at the UK Met Office and Météo-France, specifically in developing accurate prediction algorithms.

In 2020, the Met Office – a worldwide provider of weather forecasts – was awarded £1.2 billion of public funds for its next generation forecasting environment centred on a new supercomputer **[S1]**. The business case was underpinned by a major programme of work on Data Assimilation and Ensembles (DAE) designed to match forecasting algorithms to the computing infrastructure. One of the Met Office's key strategic decisions was around how to develop 4DVar and if an ensemble approach (4DEnVar) offered a promising alternative. In making such a significant change to forecasting, the Met Office drew on evidence from Surrey's research **[R1]**, stating that it "*contributed significantly to the knowledge that underpinned strategic decisions about data assimilation methods the Met Office should develop"* **[S2].**

As a direct result of the findings from the Surrey work **[R1]**, the Met Office undertook further investigations over the period 2014-2018, culminating in the internal report "*The development of a hybrid 4D-ensemble variational assimilation system*". The Met Office Expert Scientist confirmed that "*solving or mitigating the localisation problems highlighted in **[R1]** is needed to realise the full potential of 4DEnVar in the new system*" **[S2]**. The Met Office therefore, took the strategic decision to continue developing 4DVar alongside 4DEnVar **[S2]**.

The research insights gained by the Surrey research have been further exploited, via Fairbairn, at the Météo-France (2014-2017), the national forecasting agency for France and its overseas territories, where research is also "*actively addressing the issues related to 4DEnVar demonstrated in **[R1]**, including those associated with localization*" **[S3]**.

Increasing Public Understanding of Weather Forecasting

The book, Invisible in the Storm: The Role of Mathematics in Understanding Weather by Roulstone and John Norbury (Princeton University Press, 2013) describes "*how the development of mathematical ideas, combined with modern computer technology, has completely transformed our ability to understand and predict the weather*" (endorsement by Roger Penrose). Reflecting the research at Surrey **[R1-6]** it "*picks apart the challenge*" of forecasting chaotic systems, describing how DA and ensemble forecasting mitigate the butterfly effect and outlining the role of geometry in applications of PV conservation **[S4]**. The book has been translated into German (2019) and has sold over 2,000 copies worldwide (up to August 2020) **[S5]**.

A review by the European Mathematical Society stated, "*The authors have done brilliant work to collect a huge amount of historical information, as well as mathematical information, but keeping always a level in the explanations that makes the text accessible to undergraduate students in the first years, and even to people not so familiar with mathematics*" **[S6]**.

Further reviews illustrate the reach of Invisible in the Storm:
"*As a TV weather forecaster for over forty years, I have always maintained that meteorology depends on mathematics for meaning. Making this conclusive point, Invisible in the Storm takes readers on an intriguing journey through the history of meteorology, revealing the critical role of mathematics from the earliest days of weather predicating to the current age of computer-generated forecasts. This book guides you inside the storm, where maths' importance is clearly visible."* – chief weather forecaster at ABC-7 News/KGO-TV, San Francisco **[S7]**.

"I recommended Invisible in the Storm both to mathematics undergraduates and educators who are interested in applied mathematics, weather forecasting, or both." – Mathematics Teacher **[S7]**.

In 2015, Roulstone and Norbury were awarded the American Meteorological Society's Louis J. Battan Author's Award for illuminating "*the mathematical foundation of weather prediction with lucid prose that provides a bridge between meteorologists and the public*." **[S8]**

In 2016, Roulstone was a consultant for a three-part BBC Four documentary, Storm Troupers: The Fight to Forecast the Weather (BBC4, May 2019), initially watched by over half a million viewers before repeats and further distribution **[S9]**.

Producer James Sandy from Keo Films said that *Invisible in the Storm* was "*an invaluable resource*", "*both accessible and captivating*", that enables them to present weather forecasting as a "*triumph of modern mathematics and physics that stands alongside breakthroughs such as mapping the human genome*." **[S10]**

5. Sources to corroborate the impact (indicative maximum of 10 references)

[S1] Met Office Press Release. (17 Feb 2020). Up to £1.2billion for weather and climate supercomputer. https://www.metoffice.gov.uk/about-us/press-office/news/corporate/2020/supercomputer-funding-2020

[S2] Testimonial from Dr Stefano Migliorini, Manager, Next-Generation Data Assimilation Project, Met Office. (PDF)

[S3] Testimonial from Dr David Fairbairn, Scientist, Earth System Assimilation Section, European Centre for Medium-Range Weather Forecasts. (PDF)

[S4] P. Ball (2013), No hurricane tonight, Prospect 203: https://www.prospectmagazine.co.uk/magazine/weather-forecasting-climate-change

[S5] Royalty statement, August 2020: Sales figures for Invisible in the Storm. (Confidential)

[S6] European Mathematical Society. (5 June 2013). Invisible in the Storm: The role of mathematics in understanding weather. Book Review: https://euro-math-soc.eu/review/invisible-storm-role-mathematics-understanding-weather

[S7] Princeton University Press. Invisible in the Storm - Praise. https://press.princeton.edu/books/hardcover/9780691152721/invisible-in-the-storm

[S8] American Meteorological Society, Louis J. Battan Author's Award 2015: https://www.ametsoc.org/index.cfm/ams/about-ams/ams-awards-honors/awards/search-past-award-winners/

[S9] Viewing figures for Storm Troupers: The Fight to Forecast the Weather. Email from Cassian Harrison, Editor BBC4 (dated 18/11/2020).

[S10] Testimonial letter from James Sandy, Producer, Keo Films.

Book
Roulstone, I.& Norbury, J. (2013). *Invisible in the Storm: The Role of Mathematics in Understanding Weather*. Princeton University Press. ISBN: 9780691152721

Television Documentary
Sen, P. (Executive and Series Producer). (May 2019). *Storm Troupers: The Fight to Forecast the Weather* [Television series]. United Kingdom: BBC4.