Institution: University of Aberdeen

Unit of Assessment: 10 (Mathematical Sciences)

1. Unit context and structure, research and impact strategy

a. Context and structure.

The **Institute of Pure and Applied Mathematics** (**IPAM**) at the University of Aberdeen brings together leading researchers in Pure and Applied Mathematics to sustainably create and develop research ideas whilst also collaborating actively with other scientists across the breadth of academic disciplines. *Pure Mathematics* has 14 FTE and *Applied Mathematics*, associated with the Physics unit, consists of 14 FTE total, of whom 10 are included in this submission and 4 in submissions of other UoAs. The two groups, together with Chemistry, Computing Science and the remaining staff in Physics, form a larger organisational and financial unit within the University called the **School of Natural and Computing Sciences**. This structure fosters interaction and co-operation between the Pure and Applied mathematics groups and with other disciplines within the School.

The core research in the **Pure Mathematics** group is in the areas of:

i. **Topology** (7 members), including Differential Geometry, Homotopy Theory, Applied Topology and Neurotopology.
ii. **Algebra** (7 members), including Representation Theory, Algebraic Groups, and Tropical Algebra.

The Topology and Algebra groups have strong research links resulting from our research and the staffing strategies described in Sections 1b and 2b below. We enjoy an international reputation in Topology, a traditional strength of Aberdeen. The theory of *p*-local finite groups, an important link between Topology and Representation Theory (Algebra), was invented at Aberdeen (with major collaborators in Barcelona and Paris) and has been the subject of intensive research over the past two decades.

Research in the **Applied Mathematics** group centres on the following areas, with several members belonging to more than one group:

i. **Dynamical Systems** (8 members) including Systems Biology, Chaos Theory, Fluid Dynamics, Complex Networks, Epidemiology and Ecology.
ii. **Data Science** (7 members) including Data-Based Modelling, Model-Based Data Analysis, Machine Learning, Statistics and industrial applications.

One member specialises in Quantum Theory.

Interdisciplinarity is at the heart of the Applied Mathematics group, enabled through collaborations with other disciplines within and outside the University and through industrial partnerships. We have close connections to the **Institute of Medical Sciences** (**IMS**) and the **School of Biological Sciences** at the University. Our collaboration with the pharmaceutical industry should be highlighted; notable partnerships, created over the current REF period, include those with TauRx Pharmaceuticals/TauRx Therapeutics Ltd, Genting TauRx Diagnostic Centre Sdn Bdh, and GT Diagnostics (UK) Ltd, AstraZeneca, Ingenza Ltd and Fujifilm Diosynth Biotechnologies.

b. Research strategy, achievements and objectives.

The **research objectives** in this REF period, as set in our submission to REF2014, were as follows:

i. Foster areas of strength (in 2014, these included Algebra and Dynamical Systems).
ii. Create an all-round research environment by “filling in gaps” between the various research groups.
iii. Strengthen interactions between the Pure and Applied Mathematics groups to maximise opportunities for interdisciplinary research and generate impact.

iv. Create new research groups as opportunities arose, building on existing strengths.
v. Capitalise on our expertise in Dynamical Modelling and Data Analysis through existing multidisciplinary collaborations in the Life Sciences and Medicine to attract investments and expand the impact of our research.

We actively adopted this plan during the current REF period with considerable success. We obtained substantial research funding from industrial partners and the Research Councils. We ran a vigorous research programme, evidenced by publications in leading journals such as the Annals of Mathematics, Memoirs of the AMS, Physical Review Letters, Nature Communications and PNAS.

The Pure Mathematics group focused on its strength in Topology, and new appointments were made at the interface between Topology and Algebra, leading to successful collaborations (see Section 2b). A new research group in **Neurotopology** emerged within Pure Mathematics thanks to Levi's research connections with topologists at the École Polytechnique Fédérale de Lausanne (EPFL), which led to collaboration with the **Blue Brain Project** at the EPFL (**BBP**). This is a flagship multi-million-dollar project funded by the EU. Levi's research revealed unexpected topological phenomena in the brain's electric activity which received substantial attention from the scientific community. The collaboration has led to several postdoctoral positions and PhD studentships at Aberdeen (Section 3a). Levi used tools from **Topological Data Analysis** (**TDA**) in his research with the BBP. This new expertise has enabled us to implement our strategy of filling in gaps between the Topology and Data Science groups. It has yielded interdisciplinary collaborations with researchers in the IMS (see Sections 1c(iv) and 4a below).

The development of the Dynamical Systems and Data Science groups within the Applied Mathematics group has resulted in a substantial increase in the volume and variety of the impact generated by the unit since REF2014. This increase was driven by strategic appointments towards the end of the REF2014 period and focused activities within the unit during this REF period. The number of our industrial partners has grown considerably as well.

The main weight of our impact has been in the medical field (for example 1.3 FTE staff are bought out by the company **TauRx Therapeutics** (TauRx)). Impact generation has expanded beyond the medical industry to include public policy (in connection with food safety and COVID-19; see Section 1c below), impact on society via interdisciplinary research in education (Section 1d), and further collaborations with industrial partners related to cryptography. This process is ongoing, and our goal is to increase this level of activity in the next REF period.

Obtaining **external funding** is key to the sustainability of the unit. We have generated substantial income from industrial partnerships, as well as obtaining support from grant funders, including the Research Councils and the European Commission. The increase in income in comparison to the previous REF period is substantial and surpasses the sector growth for this period (see Section 3a).

Increasing interactions between the Pure and Applied Mathematics groups is important to maximise interdisciplinary research and the generation of impact. This is an ongoing process that relies on joint activities such as a series of Interdisciplinary Seminars (Section 1d below).

The **research objectives** for the next REF period are as follows:

i. Strengthen the "anchor" research areas in Topology and Dynamical Systems, building on Aberdeen's international reputation in these areas.
ii. Develop the interdisciplinarity and delivery of impact, with emphasis on applications of Dynamical Systems, Data Science and Neurotopology.
iii. Actively expand our portfolio of non-academic partners with interests in Data Science, which has become a notable area of strength of our unit, and look for collaborative funding opportunities.

iv. Strengthen the Algebra group and increase its links with other disciplines both within and outside the unit.
v. Increase interaction between the Pure and Applied Mathematics groups and develop further collaborations between them.
vi. Increase our postgraduate research (PGR) student numbers.

For (i)-(v), we will build on the measures described in Sections 2a and 2c to support research and impact generation, taking advantage of the university infrastructure described in Section 3a and in REF5a. We plan to hire new staff to support existing and emerging areas of strength (Section 2b). For (ii), (iv) and (v), we will capitalise on the interdisciplinary nature of our research (Section 1d) to expand our collaborations within the unit and with other academic disciplines. We have extensively diversified the academic areas we work in and the institutions – academic and industrial – with whom we collaborate and deliver impact. For (vi), see Section 2e(i).

A crucial prerequisite to implementing our strategy is to increase our **external research funding**. We expect our research income to grow as a result of the continuing increase in the number of interdisciplinary collaborations. We will continue to use the University's **Grants Academy,** which supports researchers in locating funding sources and applying for grants (Sections 3a and 3b), and the external organisations described in Section 4a to obtain funding from industry. Measures such as the **Annual Appraisals** described in Section 2a are in place to continually encourage staff to apply for grants.

c. Impact Strategy.

The unit has experienced significant growth in impact since REF2014, during which period appointments were made towards establishing connections with industry. An example is Schelter, whose connections with the pharmaceutical industry have led to the successful collaboration of himself and Thiel with TauRx, now a major vehicle for impact and knowledge exchange (KE) for our unit as well as for others (Biology, Chemistry, and Psychology). It was a catalyst for the engagement of others in impact-generating activities within and outside the pharmaceutical industry. This success is presented in the unit's REF2021 impact case studies, two of which are related to this research: see examples (i) and (ii) below. We implemented measures such as block teaching (see Section 2a) to allow members of staff to develop connections with industrial partners and generate impact.

Until 2014, most of the unit's impact arose from research in Dynamical Systems and Time Series Analysis. We made a strategic decision to expand to Data Science and Statistics, reflected by the appointments of Schelter (Data Science) and Vogel (Statistics) and by Levi's new expertise in TDA. This course has generated new high-impact research in areas beyond pharmaceuticals, some examples of which, involving impacts on **commerce, economy and health** via very different technologies, are showcased below. Key examples of impact include:

(i) **Data Science** research on dementia drug trials in partnership with TauRx has led to the development of novel statistical analysis tools for dementia treatment. This research contributed to providing a better understanding of the functioning of the treatment and suggested new designs for clinical trials and addressed the major concern of drop-out in these trials. [Redacted]

(ii) [Redacted]

(iii) **Enigmedia** is a Spanish company that provides industrial cybersecurity products with unique technology, enabling encryption with latencies below 1 millisecond in low-capacity processors and battery-free sensors. Research in **Cryptography** led by Baptista in collaboration with Enigmedia has allowed scrutinisation of their encryption protocols and their security, portability and performance in different chipsets. Baptista was appointed as a scientific advisor to Enigmedia, a new industrial partner of Aberdeen.

(iv) The research expertise in **Applied Topology** acquired during the present REF period allowed Levi to collaborate with the Institute of Medical Sciences (IMS) at Aberdeen on the development of a new MRI scanner to perform fast field cycling MRI (**FFC-MRI**). Levi provided TDA tools to enhance the performance of the prototype (Section 4a). In addition, Levi's research in this area led to his appointment as a senior consultant at Inait – a spin-off company of the research from the BBP.

(v) Schelter and Thiel also conducted research for Matau Beatha Ltd via the Innovation Voucher scheme run by **Interface** (www.interface.org), whose mission is to promote innovative business–academic collaborations. This example exhibits our engagement with impact generation and our responsiveness to the channels available to achieve it. The project involved mathematical modelling to facilitate the development of a device to monitor cask integrity, a crucial step in the production of whisky, an industry worth GBP4.7bn that contributes an estimated 21% of all UK food and drink exports. The models provided a feasibility study and informed the development of a prototype.

Research in the unit has also impacted public health policy, as described in our submitted REF impact case studies. The underpinning research for these examples relies on the unit's strategic strength in data analysis and machine learning.

(vi) Source attribution models alongside machine-learning methods were developed by Perez-Reche and Strachan to identify the origin of pathogens causing **infectious diseases** in food sources. Based on this research, collaboration with Forbes (medical microbiology) on the origin of human campylobacteriosis infection underpinned European regulation (EU) 2017/1495 in January 2018, which stipulates the permissible levels of campylobacter in chicken carcasses, thereby shaping food safety regulation across the European Union. This work is submitted as an ICS to UoA1, evidencing the multidisciplinary nature of our work. Further work of Perez-Reche and Strachan has challenged previous views on food sources of Listeriosis.

(vii) Modelling of the spread of infectious diseases applied to **COVID-19** has led to a contract involving Perez-Reche with the NHS and the Scottish Parliament under the Scottish Parliament Framework Award for the Provision of Research Services as an advisor on COVID-19 testing and the dynamics of its spread. We expect this research to impact public policy and health and wellbeing.

(viii) The work of Grebogi, Schelter and their collaborators on **Foetal Alcohol Spectrum Disorder** showed that no safe level of alcohol consumption during pregnancy exists. This work, featured in the journal Chaos, disproved prevailing scientific and medical opinion and was reported in the UK media (e.g., Daily Mail Scotland and Edinburgh Evening News) and noted by The Queen's Nursing Institute Scotland. It also attracted international press coverage in various countries and ranks in the top 5% according to AIP publishing's Attention Score.

d. Interdisciplinary research.

During the REF2014 period, our main area of applicable expertise was in Dynamical Systems and Systems Biology, which we harnessed to develop interdisciplinary research mainly with academics in the medical and biological sciences. During the present REF period, we have expanded the scope of our interdisciplinary research by investing in new areas such as Data Science. Consequently, the volume and diversity of the interdisciplinary research in the unit have grown substantially.

We have continued nurturing our long-standing collaborations in the area of Medical Sciences, with many collaborative projects on Systems Biology and Mathematical Biology with the IMS. An example includes an ongoing collaboration of Romano and Stansfield (IMS) on the mathematical modelling of protein production processes. Another is Ullner's collaborations with Bewick (IMS)

on modelling nerve terminal endings, with Müller and Pettitt on modelling cis/trans splicing and with McEwan on the regulation of androgen receptors. Another is Perez-Reche's work with Forbes on human bacterial pathogens. Levi is collaborating with MacLeod (IMS/NHS) and Oren (Computing Science) on the development of FFC-MRI (Section 1c(iv)).

External collaborations in medical sciences include those with the University of Exeter (mathematical modelling of combinatorial stress responses), Oxford University (mathematical modelling of DNA replication), Université de Cergy-Pontoise and the University of Osnabrueck (theoretical neuroscience), and AstraZeneca (modelling of cancer growth).

Beyond Medical Sciences, Martin and Thiel are collaborating with Martin (School of Education) and with Wolfram Research on mathematics education. They are creating tools using Wolfram's computing system Mathematica to develop the computational and problem-solving skills of learners ranging from nursery to university students and are assessing the effect this has on their learning.

Thanks to our new focus on Data Science, new collaborations in areas such as Healthcare are being established, with academics from the Aberdeen Centre for Health Data Science (ACHDS) and involving the NHS. Thiel and a PGR student are involved in the data science and machine-learning aspects of the Safe Haven Project, a secure database of historical data on patients spanning over 20 years, with the aim of providing more effective and efficient diagnostic tools to doctors. A current project focuses on kidney disease and the prediction of re-admission probabilities based on a patient's history.

We have engaged with non-academic stakeholders in the public sector and in industry. We have offered consultation to the European Food Safety Authority (EFSA), the NHS and Scottish Parliament on COVID-19 testing. We have established new collaborative projects with industrial partners: TauRx Pharmaceuticals and TauRx Therapeutics Ltd, Genting TauRx Diagnostic Centre Sdn Bhd, GT Diagnostics (UK) Ltd, Ingenza Ltd, Fujifilm Diosynth Biotechnologies, Wolfram Research, Enigmedia, Matau Beatha Ltd and Skylake Capital.

One key tool to sustain and stimulate interdisciplinary research has been the creation of a series of **Interdisciplinary Seminars**, where speakers from a wide variety of disciplines are invited to give a talk tailored to a general audience of mathematicians, pure and applied, to introduce their research and communicate problems of mutual interest. Each seminar consists of 2 talks, one from pure or applied mathematics and another from a different discipline (e.g., chemistry, biology, finance and ecology). This seminar encourages interactions between the Pure and Applied Mathematics groups, thereby implementing one of our strategic goals for collaborations within the unit; see Section 2b. Grant (Pure) and Henkes (Applied) have collaborated on the topology of soft-matter physics, and Gramain (Pure) and Thiel (Applied) are currently collaborating on modelling populations of Scottish squirrels. Similar research activities organised by the School have resulted in collaborations between mathematicians and chemists, e.g., Martin and Jaspars (Chemistry) on applications of Combinatorics to molecular structure, and with Medical Sciences, e.g., Levi and Heisler (IMS) in applications of TDA tools to medical research.

The unit supports individuals embarking on new research activities with interdisciplinary potential. Examples include Levi's involvement with the BBP. The approach was fruitful, e.g., the FFC-MRI project mentioned in Section 1c(iv) above. Another example is Gramain (Pure Mathematics), who completed a degree in Marine Biology during the current REF period (to supplement his PhD in Mathematics). The unit viewed his studies as a strategic investment and supported him as detailed in Section 2a below. Soon after completion of his degree, he began collaborating with Jamieson from the School of Biological Sciences to estimate the population density of deep-sea fish, which became a project involving two PhD students after Jamieson moved to Newcastle. Gramain has recently begun another collaboration with Fernandez (School of Biological Sciences) to develop new multi-species models for fisheries to inform harvesting

strategies and seek funding from the Science and Research Fund of the Marine Stewardship Council.

We have taken advantage of the support and key connections that the **Research and Innovation (R&I) Directorate** at the University of Aberdeen offers to establish links with industrial partners, through their **Impact and Knowledge Exchange Team**. For example, they have facilitated contact with **ONE** (Opportunities North East) and **The Data Lab** (thedatalab.com) networks that bring together academics and small to medium-sized industrial organisations (see REF5a).

e. Open access research and research integrity.

Open access and quick knowledge transfer are important to our overall strategy to achieve effective dissemination and to enable impact. It has long been a tradition in the pure mathematics community to make pre-publications publicly available through platforms such as arXiv.org, where nearly all of the unit's Pure Mathematics publications are available. It provides an environment that encourages scrutiny of work in progress and research integrity.

Over the present REF period, over 300 outputs from the unit were made available in the public domain via preprint servers such as **arXiv**, **medRxiv** and **bioRxiv**, well before their publication in print or even their acceptance for publication. Early-career researchers (ECRs) and postdoctoral research assistants (PDRAs) are encouraged to engage with open access platforms.

In addition to manuscripts, related data sets and source codes are deposited in the public domain via platforms such as figshare.com and NCBI. Among many examples, we highlight the following:

(i) The data sets and executable codes used in the research of Perez-Reche and Strachan described in Section 1c(vi), which are available from the **figshare.com** repository.
(ii) Romano's publication: RNA sequencing data deposited in **NCBI**'s Gene Expression Omnibus; source codes deposited in the **European Bioinformatics Institute** Biomodels database (www.ebi.ac.uk).

Post-publication manuscripts are deposited in the University's OA repository (green open access). Members of the unit are required, and periodically reminded, to keep the repository up to date.

Considerable growth in institutional open access policies has occurred. ECRs are encouraged to take part in training that raises awareness of the green and gold open access routes, e.g., via **Grants Academy** workshops run by Scholarly Communications Services, provided centrally by the University (see REF5a).

2. People

a. Staff support and development strategy.

Knowledge transfer, the dissemination of research and the establishment of research networks are important for researchers' professional development. The Pure Mathematics group runs a vigorous programme of three weekly **seminars** (Algebra, Topology and Applied Topology), with a good balance of internal and external speakers (typically 50%-50%), to transfer knowledge, build networks and facilitate professional development. Roughly 30 research lectures are given by external speakers every year. We encourage interdisciplinary knowledge exchange between the Pure and Applied Mathematics groups by running a joint Interdisciplinary Seminar with internal and external speakers from a variety of disciplines (see Section 1d above). We also run reading seminars for staff and PGR students; recent topics include "spherical buildings", "algebraic groups" and "homotopy groups of spheres".

The organisation of **conferences** is an important pathway to international networking. We organise a series of international conferences on Topology and Algebra on the Isle of Skye. We organised the 13th Experimental Chaos and Complexity Conference (ECC) in 2014. We will host the Dynamics Days conference in 2022. See Section 4d.

The unit normally funds one research trip for each staff member every academic year. We also provide funding and facilities to visitors who come to give seminars or do collaborative research.

We operate an **Annual Appraisal scheme** for staff in the unit, including academic staff, research assistants and support staff. The appraisal process consists of the preparation of an annual review document describing the past year's achievements and plans for the coming year. This process is followed by an informal discussion with their line manager, which provides an opportunity to reflect on the progression of the appraisee's career and progress against goals and identifies any potential areas for support. Upward feedback from the appraisees benefits line managers and informs their strategic plans.

Teaching loads are assigned in a flexible manner to allow for staff professional development. Arrangements are made for **block teaching** (typically, an entire course taught in an intensive three-week block) to allow researchers to be seconded. Thiel benefits from this arrangement while seconded to TauRx Therapeutics Ltd and Genting TauRx Diagnostic Centre Sdn Bhd. In some cases, staff are facilitated to concentrate their teaching in a single half-session so that they can make longer research visits. During the REF period, at least two members of staff (e.g., Gorbounov, Izhakian) have benefitted from this arrangement every year. In line with the institution's revised sabbatical policy, two members of the unit benefitted from sabbaticals (Kedra 2015/16, yielding 3 papers and a successful Leverhulme grant application, and Henke 2019/20, who attended a thematic programme in Cambridge). The unit arranges teaching and administrative duties to enable sabbatical leaves.

We implement a flexible approach when appropriate. For example, Gramain completed a degree in Marine Biology during the current REF period (see Section 1d). To allow him to complete his duties as a student and his responsibilities as a parent, we devised a workload scheme that minimised his timetabled commitments.

The University's Researcher Development unit (**RDU**) provides a range of **professional development** courses and workshops for researchers at all levels: from induction courses for new appointees with emphasis on support for ECRs, to courses for new PhD supervisors, to an International Leadership Development Program. Members of the unit are encouraged to take advantage of these workshops (see REF5a).

During the REF period, 19 successful internal promotions occurred, 4 to Personal Chair, evidencing the effectiveness of the strategies for staff development.

b. Recruitment strategy.

The main research areas in Pure Mathematics at Aberdeen are Algebra and Topology, the latter being a traditional strength. Recent Pure Mathematics appointments were made at the interface of Topology and Algebra to maintain existing strengths and promote interaction between research groups. Examples include Martin and Meir (Algebra) and Patchkoria (Topology). Indeed, Martin and Meir have already been collaborating and producing papers with existing and new members of the Topology group, showing the success of this strategy.

Our strategic goal to create more interaction and collaboration between the Pure and Applied Mathematics groups led to the appointment of Grant and Vogel in 2014, both at an early career stage. Grant is a topologist interested in applications to robotics, and Vogel is a statistician with a background in pure mathematics. Both are linked to our research on Dynamical Systems,

Mathematical Modelling and Data Science. Various members of the unit have broad expertise outside mathematics. For example, Romano is partly affiliated with the IMS, while Levi has expertise in Neuroscience and Gramain in Marine Biology.

c. Early-career researchers.

ECR staff are normally supported by a 3-year probationary period. We assign a mentor to support their development as researchers and teachers and assist with their integration into the system and culture of Aberdeen. Mentors are senior colleagues who provide guidance on the progression of their career and shape their training and development plans, including funding applications. Progress is reviewed annually by the mentors, similar to the Annual Appraisal scheme described in Section 2a, by means of annual reports and meetings to set up future career development goals. This process clarifies to ECRs the expectations during the probationary period. ECRs have reduced teaching and administrative loads to give them time to concentrate on research and to apply for appropriate research funding. In the first year of their appointments, the reduction in teaching is 50%, and in the second year, it is 25%.

On appointment, ECRs receive a GBP5K start-up grant to spend during the first 3 years of their appointment to support their research activities. To further help their career development, ECRs receive priority when we allocate university-funded PGR studentships. We often assign the organisation of seminars to either new permanent members of staff or PDRAs. This gives them an opportunity to develop new research networks.

We are committed to developing the career pathways of fixed-term researchers, in line with the Concordat to Support the Career Development of Researchers (see REF5a). PDRAs benefit from the Annual Appraisal scheme, and we encourage them to take part in appropriate Researcher Development training. They are given opportunities to take up teaching responsibilities, usually running tutorial sessions. In recent years, four PDRAs were given full responsibility to deliver core UG courses. One of them (Smith) has just moved on to a permanent academic job.

d. Exchange between academia and business.

Industrial mathematics is an important pillar of Applied Mathematics at Aberdeen. The interaction and exchange of ideas and people between academia and business is key to our activity and is embedded in our strategy for research and recruitment.

Most of our Applied Mathematics staff are affiliated with an industrial partner or partly affiliated with other research areas of the University. For example, Schelter and Thiel are seconded to TauRx Therapeutics Ltd and Genting TauRx Diagnostic Centre Sdn Bhd, and Levi is a senior consultant to Inait. Measures are taken in terms of teaching and administrative loads to enable these exchanges (e.g., block teaching; see Section 2a above).

The flow of knowledge and ideas from businesses to stimulate academic research is a pathway we exploit. We rely on existing industrial connections but are also very active in taking advantage of platforms to establish new connections supported by the University, such as the Researcher Development Unit (RDU) (Section 2a, REF5a). For example, an interdisciplinary project by Romano and Stansfield (IMS) on recombinant protein synthesis originated from a meeting organised by a knowledge transfer network, in which they made contact with Ingenza Ltd. Similar contacts with smaller enterprises resulted in research at the unit inspired by applications (see Section 4a for some details of these projects).

Wolfram Research interacts with us in a variety of ways, ranging from joint work with Genting TauRx Diagnostic Centre Sdn Bhd on industrial prototypes of diagnostic tools to advising on our teaching programme content to make it relevant outside of academia. Staff from the unit benefit

from the conferences organised by Wolfram Research. Thiel supervises research projects at the Wolfram Summer School.

e. Research students

(i) *Recruitment strategy*
We support our students to ensure high completion rates with high-quality outcomes (see (ii) and (iii) below). The School normally provides 1-2 funded PhD studentships each year. Other strategies to obtain and fund postgraduate studentships include academic partnerships such as the University's strategic partnership with Curtin University in Perth, Australia (the **Aberdeen-Curtin Alliance**), through which we currently have 3 students.

(ii) *Monitoring, personal development and support.*
The main central source of support for PGR students is the **Postgraduate Research School**. They arrange support and training for PGR students such as the induction sessions discussed below and deal with the formalities surrounding thesis submission and examination (see REF5a).

Every student is allocated a lead supervisor and a second supervisor. Throughout the PhD programme, each student's **progress is monitored** by a committee consisting of their supervisors and an independent member of staff. Students must report annually by submitting a written report and by giving a seminar-like presentation to the committee. Progress is monitored, and goals for the next stages are proposed. The process also prepares students for defending their theses at viva.

All new PGR students attend a two-day **induction programme** covering all aspects of postgraduate education including safety, IT facilities, undergraduate demonstrating, relationships with supervisors, and the monitoring and support programmes. In addition, the University offers an induction for new supervisors to make them aware of their responsibilities and of the ways they and the University can support their students from start to completion of their theses.

The unit has a range of activities to support students in their professional development from the early stages to completion. Examples include the following:

(1) **SMSTC** (Scottish Mathematical Sciences Training Centre) is a collaborative Taught Course Centre that provides training for PGR students in Mathematics. The courses are aimed at students who have just completed their bachelor's degree. Students normally take courses relevant to their research. Both Pure and Applied Mathematics students attend these courses, with the Pure group regularly contributing to the teaching of the Topology and Algebra courses.
(2) **SUPA** (Scottish University Physics Alliance) is another collaborative Taught Course Centre for PGR students. SUPA offers Applied Mathematics courses such as Mathematical Modelling and Data Analysis. They also cover a range of areas from Mathematical Physics, such as Gauge Theory and Statistical Physics. In addition, a range of Professional Development Trainings are offered.
(3) We encourage students to meet and collaborate with students from other institutions. Students in Pure Mathematics take part in "**The Burn**", an annual two-day-long residential meeting of PGR students across Scotland sponsored by the International Centre for Mathematical Sciences. Students present their work as conference lectures and can collaborate and enlarge their networks.
(4) Students are encouraged to participate in the research seminars and reading seminars we run. Many of our students have given research talks in the Algebra and Topology seminars and even organised some of our **reading seminars**.

(iii) *Skills development and preparation for their future careers.*
The unit covers the travel expenses of PGR students to one conference every year to help establish research networks and disseminate their research.

PGR students are given teaching opportunities as teaching assistants, with hours they are allowed to teach limited to avoid hindering their research.

In this REF period, in both Pure and Applied Mathematics, 75% of the students who completed their PhDs were able to find academic jobs as PDRAs at other universities. This success evidences the effectiveness of our monitoring and career development activities for PGR students.

f. Equality and Diversity.

As a unit, we aim to provide an environment that allows researchers to thrive and maximise their potential. The REF5a describes the overall institutional approach to equality and diversity and sets out our commitment to inclusivity in teaching and research across the institution. This section focuses on measures specific to the unit and complements those implemented at the institutional level.

In line with mathematics departments elsewhere, most researchers in this unit are male (91%), compared with 80% across the UK sector (HESA 2018/19). 20.8% are BAME, which is slightly higher than the percentage of BAME researchers within the total REF eligible population at Aberdeen (16.8% BAME) or the HESA data (15.9% across mathematical sciences in the UK). Our staff composition is culturally diverse and includes individuals from Western and Eastern Europe, the Middle East, the Asia-Pacific region, South America and the UK. Most new staff hired during the current REF period are not British and joined the University of Aberdeen from institutions outside the UK. 62.5% of submitted staff are aged 35-49. Appointments are normally full-time, so work-life balance and caring commitments are important issues. We ensure gender representation on hiring committees, particularly for shortlisting. The School established a “Portrait Gallery” next to the main office depicting approximately 20 scientists and mathematicians from diverse backgrounds.

The research environment within the unit must take account of discipline-specific requirements alongside the specific needs of various groups of researchers. Academic duties are assigned taking into account parental and caring commitments by offering flexible working hours, including the ability to work from home part-time with adjusted teaching duties. For example, Hepworth worked from home one day a week for one year for childcare reasons; Romano’s workload excluded mandatory on-campus duties for one term upon returning from her maternity leaves. These measures help individuals with caring responsibilities and eases the transition back to active research. The School has a carer fund to help those with caring responsibilities to travel to conferences (up to GBP300).

Our PGR and PDRA populations include people from Kenya, Nigeria, Saudi Arabia, India, Thailand, Indonesia, China, Eastern and Western Europe, and the UK. This diversity is in line with our international outlook and our ambition to attract talented ECRs from across the world. We welcome and foster this diversity and encourage staff, students and postdocs to benefit from it by organising social activities (pre-seminar teas and our “cake seminar”).

Female representation among PGR students is approximately 35% (25/71 in the REF period; this count includes both completing and ongoing studentships), which is well above the average for UK Mathematics PhDs (20% according to an LMS report “Advancing Women in Mathematics” lms.ac.uk/sites/lms.ac.uk/files/LMS-BTL-17Report_0.pdf). Of those who completed their PhD, 23% were women, above the sector average (19% according to the same report). Of the PDRAs in Applied Mathematics who left Aberdeen for a permanent academic position, 50% were women. An indication of our supportive and inclusive environment in

research is evidenced by 25% of our seminar speakers (in the period from January 2019 to present) being women, serving as role models and encouragement to our female students. The unit supported the participation of one female member of staff (Romano) in a series of events organised by Aurora (Advanced HE's leadership development initiative for women) prior to her promotion to personal chair.

Training on **Equality and Diversity** legislation and policies is compulsory for all staff sitting on selection or promotion panels and all line managers, supplemented by **unconscious bias** training.

In preparing this submission and selecting outputs and impact case studies, the unit has complied with the requirements of the institutional Code of Practice. Outputs were selected based on quality, as indicated by at least two independent reviews and in accordance with the principles of the San Francisco Declaration on Research Assessment (DORA). All colleagues were invited to declare circumstances that could entitle the unit to a reduction in the number of outputs required.

The School of Computing and Natural Sciences holds an **Athena SWAN bronze award** (2016).

g. The Concordat

We are committed to the Concordat to Support the Career Development of Researchers. Special induction sessions on HR and E&D regulations and policies are compulsory for new appointees (Principles 1-4 p.2 and p.4(2) of the Concordat). REF 5a describes the structures that support our postdoctoral researcher population and testify to our commitment to implement the Concordat. Our PDRAs take part in our Annual Appraisal scheme (Section 2a) to aid in their career development. They are encouraged to become seminar organisers and gain teaching experience with us by delivering lectures and tutorials. They are invited to participate in the Researcher Development programmes offered by our RDU to further their career prospects. During the REF period, 2 PDRAs from Pure Mathematics and 8 from Applied Mathematics secured permanent lectureship positions (examples include Smith at Nottingham Trent University, Lynd at the University of Louisiana, Elsegai at Cairo University, Morales at UC Davis, and Rubido in Uruguay), and 11 secured temporary academic jobs. Teaching fellows benefit from these support structures as well, and one of them secured a permanent lectureship (Elmer at Middlesex University London), see REF5a.

3. Income, infrastructure and facilities

a. Generation of research income.

The unit experienced a substantial increase in research income during this REF period. Per FTE income per year increased from GBP26.4K in REF2014 to GBP39.2K in REF 2021, an increase of more than 45%, significantly higher than the overall sector growth of research income recorded in HESA for this assessment period (13%). The sectoral share has risen from less than 0.05% in 2013-2015 to 0.34% in 2017-2019.

Obtaining external funding is vital to the sustainability of the unit. During this REF period, most of our income arose from interdisciplinary research. We generated income from **industrial partnerships**, including GBP2.4M from TauRx Therapeutics Ltd and Genting TauRx Diagnostic Centre Sdn Bhd (Schelter and Thiel). Overall research income from industry has grown from GBP47k for the REF2014 period to GBP2.6m for REF2021, thanks to engagement and impact strategy. Further funding was obtained from Research Councils, including the **BBSRC** (notable grants: Romano GBP329K), the **EPSRC** (notable grants: Levi GBP874K) the **Leverhulme Trust** and the prestigious **European Commission** (notable grants: Politi GBP10K and Henke GBP80K for a Marie Curie Fellowship). Many staff took research trips to other universities or research

institutes such as Mathematisches Forschungsinstitut Oberwolfach, funded directly by their hosts.

Everyone is encouraged to apply for grants, and staff – especially ECRs – take part in workshops run by the Grants Academy that give advice and training in writing grant proposals. We have an established **internal peer review system** in the unit. Grant proposals are reviewed by experienced colleagues who give feedback to the applicant, thus raising the quality of the proposal and improving its success prospects. The overall research income of the unit in the present REF period is GBP942K per year on average, with only slight fluctuations, an indication of stability.

b. Organisational, operational and scholarly infrastructure to support research and impact.

The unit enjoys generous accommodation especially designed for its purpose. Apart from ample office space, it contains two large social areas for informal discussions and interdisciplinary dialogue. There are two purpose-built **seminar rooms** equipped with state-of-the-art technology (approx. cost GBP6.5K) to record, broadcast and receive lectures. Together with the social area, this forms a small-scale conference centre. The buildings and the rooms have accessibility provisions for the disabled, including lifts, toilets and wide entrances. The unit has access to a large electronic journal collection as well as printed journals through the recently built (in 2011, GBP57M) **Sir Duncan Rice Library**; this includes millions of articles, chapters and books via "Springer Link".

The University has dedicated staff within Research & Innovation and **Public Engagement with Research Unit** (PERU) to identify funding and public engagement opportunities and communicate them to relevant academic staff. The **Grants Academy** supports project development, networking and grant writing. **Research Financial Services** provides advice on costing, preparation and administration, along with the production of final reports. Our staff make ample use of these services (see REF5a). The work with Matau Beatha Ltd (Section 1c(v)) was a direct consequence of the support from R&I. The recent grant application of Gramain, joint with the School of Biological Sciences, was in response to communication from the R&I unit.

All researchers have ample central storage for data (2TB per project, extendible if required). Support is available centrally, with data management and curation provided.

Key to our impact delivery is Aberdeen's **High Performance Cluster** (1240 cores and 12 TB of RAM, see REF5a). It is used for data analysis by the Applied Mathematics group for their research in nonlinear time series analysis and mathematical modelling. Individuals have access to GPU computing nodes and to a wide range of software packages, including Mathematica and MATLAB.

The University's **Impact and Knowledge Exchange Team** provides support for impact-related and industrial activities as well as IP management. They have managed Schelter's contracts with TauRx Therapeutics Ltd and Genting TauRx Diagnostic Centre Sdn Bhd.

PERU facilitates many of our outreach activities (Section 4c). The Research Policy and Strategy team in R&I provides high-level oversight of research governance. The University's ethics committees manage the ethical approval process for research with human participants or involving other ethical issues. Martin was Convenor of the Physical Sciences and Engineering Ethics Board for 2.5 years and contributed to the latest update of the Research Governance Handbook. Online training in **research integrity and ethics** is compulsory for staff. Data storage and privacy/GDPR are handled by **IT Services** and the University's **Data Protection Officer**. They advise us on the many issues arising from our research in Data Science (see REF5a).

The School has recently appointed an academic **Impact Champion** (Romano, Applied Mathematics group) to promote and support impact-generation activities. Romano has been working closely with the University Impact Manager in planning a series of activities to support and inform researchers in the School about impact opportunities. The first such event took place in February 2020 and was well attended and well received.

c. Cross-HEI infrastructure.

The Neurotopology group and its PGR students have access to the **BBP High-Performance Computer** at the École Polytechnique Fédérale de Lausanne (EPFL). The protein synthesis modelling group was awarded access to **ARCHER**, the UK National Supercomputing Service. PGR students benefit from access to the Scottish Mathematical Sciences Training Centre (SMSTC) and SUPA, the infrastructure of which is provided by the networks of the universities that run them. This infrastructure includes IT communication services and administrative staff that coordinate the programme.

4. Collaboration and contribution to the research base, economy and society

a. Research collaborations, networks, partnerships and impact.

The unit collaborates nationally with many universities including Reading, Manchester, Exeter, Oxford, UCL, Edinburgh and York. International collaborations include universities in Switzerland, Spain, Germany, Brazil and the USA. Of particular importance is the network of collaborations in China established by Grebogi, who co-founded the **Aberdeen-Lanzhou-Tempe Joint Research Centre**, dedicated to interdisciplinary research. The network includes Fudan, Xi'an Jiaotong, Northwestern Polytechnical, Tianjin, Beijing Normal and others. As a result, the unit has benefitted from over 30 long-term visitors, many from China. Some of these visitors were supported by external funding from the Leverhulme Trust and the SUPA Distinguished Visitors Program.

The University was one of eight partners in the Innovative Training Network COSMOS (Complex Oscillatory Systems: Modeling and Analysis), active from 2015-19. In 2019, COSMOS presented their results at the exhibition Going Global in Berlin.

For industrial partners, see Sections 1c(i)-(iii). To expand our network of connections, we liaise with the R&I (Section 3b) and with Scottish innovation centres that work with private and public organisations to accelerate innovation enterprises, e.g., Opportunity North East, The Data Lab, Interface and Censis. They facilitate contacts between academics and non-academic organisations and provide funding for collaborative projects. The collaboration of Schelter and Thiel with Matau Beatha Ltd (Section 1c(v)) was a direct consequence of this support.

The Interdisciplinary Seminars and other activities to disseminate research, such as research days run by the School, facilitate collaboration with academics across the University. An example is Levi's collaboration with MacLeod from the IMS at Aberdeen on the FFC-MRI described in Section 1c(iv), which began thanks to one of the research days, in which MacLeod learned about TDA. The FFC-MRI is a multidisciplinary project combining state-of-the-art Physics, Artificial Intelligence and Data Science. It is associated with a GBP300K grant awarded to MacLeod to build a prototype and a proof of concept with a view to attract investments. Funding has also been obtained for a 12-month PDRA to work on the project with Levi. This project demonstrates how we translate networking to further academic collaborations and funding.

We take advantage of teaching activities as well to extend our research networks. Since 2016, the unit has offered an MSc programmes on **Financial Mathematics** and **Data Science**. Connections we made with Bentley University (USA) to design the curriculum of our programme

soon turned into a research collaboration between Kedra and Libman from Aberdeen and Steblovskaya from Bentley.

b. Wider contribution to the economy, society and academic research.

Since REF2014, research undertaken by staff in this unit has made significant contributions to TauRx Therapeutics Ltd drug development and diagnostics programmes. TauRx Therapeutics Ltd is a University of Aberdeen spin-out; it has secured [Redacted] of venture capital to pursue the development of therapies for Alzheimer's disease and other dementias. Genting TauRx Diagnostic Centre Sdn Bhd, TauRx Therapeutics Ltd's joint venture company with Malaysia's Genting Berhad group, is using a variety of different technologies to develop innovative tools and systems required to permit the early detection, diagnosis and monitoring of cognitive impairment related to dementia, such as Alzheimer's Disease or Frontotemporal Dementia, as well as other related disorders. GT Diagnostics (UK) Ltd has been founded by Genting TauRx Diagnostic Centre Sdn Bhd as a UK based subsidiary with their offices in Aberdeen to facilitate local research and product development activities. Research from several disciplines at the University underpin TauRx Therapeutics Ltd's, Genting TauRx Diagnostic Centre Sdn Bhd's and GT Diagnostics (UK) Ltd's work, including medical sciences, chemistry, and psychology in addition to mathematical sciences.

Recently, and in response to the global pandemic, Perez-Reche's work on COVID-19 has led to a contract with the NHS and the Scottish Parliament, with Perez-Reche serving as an advisor on COVID-19 testing. He submitted a report on 18 May 2020.

The BBP, to which Levi contributes, is at the forefront of international research to transform our understanding of the brain and its functions.

All our staff are involved in peer reviewing for mathematics and physics journals, and many review grant applications. Members of the unit are or have recently been editors or associate editors for journals including *Proc. Edinburgh Math. Soc. Series A., J. Australian Math Soc., Advances in Mathematics, Physical Review E, Chaos and Nonlinear Biomedical Physics.*

Our outreach activities (see Section 4c below) raise awareness and improve the understanding of mathematics among the general community.

c. Engagement with diverse communities and E&D.

The unit engages with diverse communities both through its research and events organised for the general public, examples of which include the following:

We run **Continuing Professional Development** sessions for secondary school teachers each year. We offer refresher sessions on topics from senior school syllabi and provide suggestions for classroom activities.

The unit coordinates the north-eastern section of the **Scottish Maths Challenge,** a yearly competition for school children. We organise and host a closing prize-giving ceremony with a maths lecture suitable for a general audience. The Maths Challenge encourages interest in mathematics and attracts a significant proportion of female participants.

Baptista and Thiel regularly host pupils for **Nuffield Research Placements**, summer projects organised by the Nuffield Foundation that help students from disadvantaged backgrounds engage in science and mathematics (nuffieldfoundation.org/students-teachers/nuffield-research-placements).

Several staff engage in outreach activities as part of **Maths Week Scotland**, **Techfest** (e.g., Perez-Reche's talk "Fighting against epidemics with mathematics"), **Mayfest** and **Science after Dark**. Martin and Patchkoria have helped design maths activities for children at the **Aberdeen Science Centre**.

Thiel uses Mathematica software tools from his project discussed in Section 1d at schools and nurseries to illustrate the power of mathematics in a fun and accessible way.

d. Contribution to the discipline's development and sustainability.

Levi's seminal publication in Frontiers in Computational Neuroscience (submitted to this REF) established a new research theme within Neurotopology that has both attracted funding and resulted in further research in mathematical biology. More importantly, the mathematical ideas of this research are simple enough to be accessible to neurobiologists, thus generating more interaction between mathematicians and biologists and opening new avenues for interdisciplinary research.

In 2001, we established the Isle of Skye **International Conference in Algebraic Topology**, which usually takes place every 4 years and is a week-long event. The latest event was in 2018, with over 100 participants and support from the LMS (GBP6K) and GMJT (GBP4K).
We will organise the 41st meeting of the prestigious **Dynamics Days** international conference in 2022. We have hosted various conferences, including the 13th Experimental Chaos and Complexity conference (2014), attracting more than 100 international participants, the Workshop on Physics and Biology of Active Systems (2015) and the Networks: from Structure to Function conference (2019).

We participate in the organisation of Scottish networks such as the Scottish Topology Seminars (2019-20), Algebra and Representation Theory in the North (2018), the Groups, Generalisations and Applications network (2017-19), the UK network in Applied Algebraic Topology (2015-16), and the British Mathematical Colloquium (2020). The Applied Mathematics group has also been involved in the co-organisation of various events worldwide, e.g., Advances in the Collective Behaviour of Complex Systems (Potsdam, 2016), New Insights in Complex Neural Dynamics (Cergy-Pontoise, 2018), and the Workshop on Ergodicity Breaking in Many Body Systems (Natal, Brazil, 2018).

We contributed to the SUPA and SMSTC networks by coordinating and contributing to the teaching of several modules, e.g., 'Topology' and 'Algebra' (SMSTC, several contributors) and 'Mathematical Modelling and Data Analysis' (SUPA, Thiel as contributor).

We are involved with various professional bodies, e.g., we arrange an annual Edinburgh Mathematical Society lecture in Aberdeen. In January 2016, Libman took part in an EPSRC event to inform EPSRC's Portfolio Managers in mathematical sciences (pure mathematics) about engagement strategies and shaping the funding strategies and schemes for the mathematical sciences.

e. Indicators of influence.

David Benson was elected Fellow of the Royal Society of Edinburgh (2017). He has also written a book on Representations of Elementary Abelian p-Groups and Vector Bundles (included in this submission).

Celso Grebogi was Thomson Reuters Citation Laureate (2016) and Clarivate Citation Laureate (2019). He was elected Member of the Academia Europaea and was awarded a Doctor Honoris Causa at Le Havre Normandie University, France (2014), and Honorary Professorship at various universities in China: Xi'an Jiaotong University (2015), Tianjin University (2018), and Huaqiao

University (2018). He is an Honorary Expert at the National Institute of Metrology, Yantai, China (2019).

Ran Levi's publication in the high-impact journal Frontiers in Computational Neuroscience has over 307,000 views to date. It has generated considerable public interest and media attention, e.g., Levi's BBC interview (20 June 2017) and a Cover Story in the New Scientist (September 2017).

Bjoern Schelter has become the Data Analytics and Biostatistics Lead of the company TauRx Therapeutics Ltd and CEO of the company Genting TauRx Diagnostic Centre Sdn Bhd.

Marco Thiel won the Wolfram Innovator Award (2017) for his innovative uses of the Wolfram technology stack.