

Institution: University of Sheffield
Unit of Assessment: B-10 Mathematical Sciences
1. Unit context and structure, research and impact strategy

1.1. Overview

Research in mathematical sciences is focused in the School of Mathematics and Statistics (SoMaS), with wide-ranging collaborative interactions throughout the University. Our aim is to create a supportive and vibrant research environment for all members of the School, at all levels. We conduct excellent research in both the foundations and applications of mathematics and statistics, harnessing the interdependence and synergies of different approaches to build and maintain strong effective intra- and inter-disciplinary collaborations.

Research is organised in six groups:

- Algebra, Geometry and Topology.
- Number Theory.
- Mathematical Biology and Environmental Dynamics.
- Solar Physics, Plasma Dynamics and Fluid Dynamics.
- Mathematical Physics.
- Probability and Statistics.

SoMaS is also home to the **Statistical Services Unit (SSU)** — a specialist advisory unit that provides comprehensive statistical consultancy services (see Section 3).

Highlights during the assessment period include:

- Significant investment in **Algebra, Geometry and Topology** to create a world-leading research group and long-term legacy for SoMaS. Capitalising on Bridgeland's election to FRS in 2014 and ERC Advanced Grant (€1.5M), we recruited four permanent members of staff in this area.
- Continued development of research impact, exemplified by Quegan's NERC Economic Impact Award (2018) for his critical leadership in the BIOMASS satellite mission, and Heaton and Blackwell's statistical contributions to the IntCal radiocarbon dating calibration programme.
- Strengthening interdisciplinary research in **Statistics** and **Mathematical Biology and Environmental Dynamics** by recruitment of four permanent members of staff in this area.
- Guaranteed permanent positions for long-term research fellows, helping us to attract and retain the best talent. This has enabled us to appoint Manolache (Dorothy Hodgkin Fellowship) and Brini (EPSRC Early Career Fellowship) in **Algebra, Geometry and Topology**, and Gielen (Royal Society Fellowship) and Campbell (EPSRC Early Career Fellowship) in **Mathematical Physics**.

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1.2 Unit structure and research strategy

The unit consists of 51 Category A staff (48.5 FTE), of whom 16 are professors, 4 readers, 15 senior lecturers, 14 lecturers and 2 senior research fellows. This represents growth of 43% since REF2014. All SoMaS Category A eligible staff have been submitted.

Our overarching objective is to develop and support focused, dynamic groups of researchers in our six areas; to generate world-leading excellence in foundational and applied mathematics and statistics; to develop new creative interactions between areas; and to use our quantitative expertise to forge collaborations that address important scientific, medical, technological and societal challenges.

Our strategy is implemented collectively by research groups and the SoMaS and Faculty of Science Research Committees, ensuring effective coordination, good practice, and identification of new collaborative opportunities. Our four key mechanisms for delivering strategy are:

- **Recruitment:** 14 new permanent appointments during the assessment period, maintaining and growing strengths and developing new areas.
- **New initiatives:** new interdisciplinary MSc programme in Mathematical and Theoretical Physics (see section 2.1); new broad “Modelling and Simulation” colloquium; statistical research consultant post (see Section 3).
- **Resource provision:** generous funding streams for research visits and collaborations (through our Mathematics and Statistics Research Centre research stimulation fund — see Section 3); regular study leave to stimulate research and impact; expansion of our University Teacher team to contribute to teaching delivery and development, releasing research time for Category A staff.
- **Funding capture:** supporting staff in grant writing and management; diversifying established funding routes with emerging opportunities (e.g. Wilkinson’s GCRF award—see Section 4a).

In the following, we outline the research focus and examples of significant achievements of each group during the assessment period. Many staff have secondary affiliations, in addition to the primary affiliations indicated.

Algebra, Geometry and Topology

[Bavula, Bridgeland, Brini, Johnson, Katzman, Manolache, Mitchener, Shinder, Strickland, Whitehouse, Willerton]

Since REF2014, we have fulfilled our strategic objective to strengthen our excellence in Algebraic Geometry by appointing Johnson and Shinder to lectureships and two fellowship holders to senior lectureships (Manolache – Dorothy Hodgkin Fellowship, £433K); Brini – EPSRC Early Career Fellowship, £1.2M). Bridgeland’s ERC Advanced Grant brought four RAs to Sheffield. Bridgeland was elected FRS in 2014 and received a prestigious Wolfson Research Merit Award. He was awarded a Royal Society Research Professorship, starting in early 2021. Whitehouse is the grant holder and organiser for the Transpennine Topology Triangle (TTT) – a long-standing regional topology seminar, supported by the London Mathematical Society (LMS). TTT is a key focus of activity for the UK topology community, and has held 111 meetings. Whitehouse is one of the guest editors of a special issue of *Topology and its Applications*, which will form the proceedings of the 2019 meeting. Professor Izak Moerdijk, who has a fractional appointment at Sheffield and has

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his substantive post at Utrecht University, has strong links with this research area. He is an organiser of the European Autumn School in Topology (EAST), an annual event since 2016, mainly aimed at PGR students/early career topologists, attended by numerous Sheffield students.

Research highlights include: relation of stability conditions to spaces of quadratic differentials, building a bridge between homological algebra and flat surfaces; solving the long-standing problem of computing the group of derived symmetries of a K3 surface; showing that stable rationality of varieties specialises in regular families, another long-standing problem; initiation of the study of L-equivalence of varieties and its relation to derived equivalence; a derivation of an algorithm to produce explicit formulas for generating functions of double Hurwitz numbers, which, inter alia, allowed for a new proof that double Hurwitz numbers are piecewise polynomial; work in homotopical algebra giving a new family of model structures related to spectral sequences; a study of the F-module and D-module structures of local cohomology modules, the study of singularities in prime characteristic, establishing the closedness of the support of local cohomology modules over hypersurfaces; a systematic study of rings of Frobenius operators; the development of effective algorithms to compute local cohomology in prime characteristics; influential contributions to work at the interface of commutative algebra and stable homotopy theory; development of a substantial programme to provide algebraic models for rational equivariant stable homotopy categories; work on adelic models of tensor triangulated categories.

Number Theory

[Berger, Dummigan, Jarvis, Manoharmayum, Şengün]

This area has been strengthened by appointing Şengün to a lectureship. Berger secured an EPSRC grant (2017–2020; £331K) to prove the modularity of abelian surfaces, one of the major challenges in linking algebraic geometry and automorphic forms. This grant funded one postdoctoral researcher. Dummigan gave a lecture course in 2019 at the National Centre for Mathematics, Mumbai. Şengün visited the Max Planck Institute, Bonn, in 2018 for a month and delivered lectures at the ICMAT mathematics programme on L2-invariants and their analogues in positive characteristic (Madrid, 2018). He was awarded an EPSRC New Horizon grant, starting in early 2021.

Research highlights include: general proof of the relation of the congruence number to the number and depth of congruences of automorphic forms; establishment of one of the first cases of the Paramodular conjecture for abelian surfaces; a detailed study of a very general conjecture on Eisenstein congruences, supported by numerical examples obtained by several means; numerical demonstration that homology of arithmetic groups can possess large torsion and provision of theoretical evidence that growth of torsion homology of arithmetic groups is deeply related to cycle complexity of the associated local symmetric spaces; proof that, in general, subgroups of GL_n over a complete Noetherian local ring with residually large image cannot be small but must contain a copy of the special linear group over the Witt ring of the residue field.

Mathematical Biology and Environmental Dynamics

[Best, Chakrabarti, Fletcher, Hawkins, Monk, Potts, Quegan]

This group has been strengthened significantly by three appointments: Potts and Fletcher to lectureships (Fletcher through a Vice-Chancellor's Fellowship); Chakrabarti to a senior lectureship. The group has strong links and collaborations with biological sciences at Sheffield. Quegan won the 2018 NERC Economic Impact Award for his substantial contributions to Airbus winning the £192M contract to build the BIOMASS satellite. He led the international team proposing and

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supporting the mission and developed many of the scientific and technological arguments underpinning it. His research has been supported by a variety of funders (see Section 3). Best was awarded a Leverhulme Early Career Fellowship, Hawkins an EPSRC First Grant (2014–2016, £99K), Potts a NERC grant (Co-I: £601K total) and an EPSRC grant (£350K); Fletcher secured a BBSRC grant (Co-I, £481K total); Monk an EPSRC IB Catalyst grant (Co-I; £932K total). Visiting fellowships were awarded to Chakrabarti (Max Planck Institute for the Physics of Complex Systems, 2019) and Monk (Konrad Lorenz Institute, 2017). Best has been awarded a Fulbright Visiting Scholarship (delayed to 2021 due to COVID-19).

Research highlights include: development of new computational methods to analyse the dynamics of embryonic tissues; modelling of the evolution of CRISPR-Cas immunity; theoretical study of integrated step selection analysis and its predictive power for animal movement; solving the longstanding problem of how radar system imperfections control the accuracy of estimates of biomass and Faraday rotation; development of a microscopic model of a disordered viscoelastic active solid, which allows to study of the collective behaviour of contractile active elements and how their interactions determines the macroscopic properties of the active material; development of a theoretical framework to understand dynamic morphologies of droplet interface bilayers when they become unstable, capturing dynamics observed in experiments.

Solar Physics, Plasma Dynamics, and Fluid Dynamics

[Ballai, Jain, Li, Mole, Ohkitani, Rees, Ruderman, Verth, von Fáy-Siebenbürgen (publishes as Erdélyi), Willis]

This group has productive links with the Automatic Control and Systems Engineering department (Balikhin, Fedun – joint funding and PhD students). Group members had observation time at external facilities such as the Swedish Solar Telescope and the Dunn Solar Telescope (New Mexico). Research in solar physics and plasma dynamics is supported by an STFC consolidated grant (£664K, 2015–2019; £998K, 2012–2016; recently renewed until 2024). Ohkitani secured £313K from EPSRC to study the statistics of fluid turbulence, Willis was awarded £270K from EPSRC for his research on “Optimisation in fluid dynamics”; Ballai received a Leverhulme Trust International Networking Grant (£76K allocated to Sheffield), the locations of the research activity being spread over the network members (Sheffield, St Andrews, University de les Illes Balear (Spain), Instituto de Astrofisica de Canarias (Spain) and University of Graz (Austria)). The group hosted eight RAs.

Research highlights include: discovery of vortex propagation across the solar atmosphere; discovery of MHD oscillations in cooling loops; the first calculation of the damping rate for axisymmetric MHD waves in a weakly twisted magnetic flux tube; a study improving the understanding of the stability of solar prominences and a demonstration that partially ionised plasma describing prominence dark plumes becomes unstable only in a two-fluid model; a study of the Navier-Stokes (NS) equations and their dynamically-scaled counterpart, assuming the former's solution breaks down in finite time: it is found that they are close to each other, which in turn constraints the possible breakdown of the solution of the NS equation; a demonstration that for pipe flow distortions appropriate to the velocity profile lead to a complete collapse of turbulence and that friction losses are reduced by as much as 90%; use of the Constrained Multiscale Turnover Lagrangian Map method to synthesize two Kolmogorov flows where persistent large scale structures produce nontrivial mean flow statistics and local anisotropy at small scales.

Unit-level environment template (REF5b)***Mathematical Physics***

[Campbell, Dolan, Gielen, van de Bruck, Winstanley]

Research in this group covers gravitation, classical and quantum cosmology, quantum field theory in curved spacetime and quantum information theory. Dolan, van de Bruck and Winstanley are supported by an STFC consolidated grant (since 2011, recently renewed until 2023), held jointly with the Universities of Manchester and Lancaster, which funded two RAs in Sheffield. Since REF2014, we have recruited Gielen (Royal Society URF (since 2018, £423K)) to strengthen this research area, who was awarded the second Buchalter Prize in Cosmology (2017). Dolan was funded by an EPSRC First Grant (2015–2017, £90K) and Campbell was supported by an EPSRC Early Career Fellowship (2015–2020, £676K).

Research highlights include: a study of the qualitative features of black hole shadows of two extremely charged black holes in static equilibrium, showing that they are examples of chaotic scattering; an investigation of the stability of black holes in Einstein-scalar field theory, arguing that stable hairy black holes represent the end point of the superradiant instability in electrically charged black holes in a cavity; developments of mathematical techniques to study quantum cosmology within group field theory; investigations of observational consequences of a variety of novel dark energy models which predict violations of the equivalence principle in the dark matter sector; work which proposes a simulation algorithm to be executed on a quantum computer that makes use of a novel random sampling technique: the first quantum algorithm with a runtime proved to be independent on the number of interactions in the system being simulated, instead the runtime depends only on the combined strength of the interactions.

Probability and Statistics

[Applebaum, Blackwell, Freeman, Geldhauser, Heaton, Jordan, Juarez, Oakley, Stephenson, Stillman, Triantafyllopoulos, Walters, Wilkinson]

Since REF 2014, we have significantly strengthened this area by making four appointments (Freeman, Geldhauser and Stephenson to lectureships; Wilkinson to a senior lectureship, promoted to chair in 2018). The group has an outstanding track record in interdisciplinary research, knowledge transfer and collaboration outside academia, with close links to the Statistical Services Unit (SSU, see Section 3). PhD students are funded not only by the standard streams available in SoMaS but also by working in partnership with, for example, Microsoft Research, Total, Historic England and the British Antarctic Survey. In 2014–17 this group had a Knowledge Transfer Partnership project with HR Wallingford (an independent civil engineering and environmental hydraulics organisation). Heaton and Blackwell serve on the international working group on radiocarbon calibration curves (IntCal). Heaton is the lead author on the Marine20 radiocarbon calibration curve covering the global oceans. Blackwell and Heaton received Leverhulme Research Fellowships to support their work on wildlife movement modelling (Blackwell) and radiocarbon dating (Heaton). There are close links with the Applied Probability Trust, which publishes two major international journals in the field and provides funding for a PhD studentship.

Research highlights include: a new approach to modelling animal movement, based on an Ornstein-Uhlenbeck diffusion process in a high-dimensional space; a first quantitative determination of the response time of North Atlantic climate to changes in high-latitude North-Atlantic-Deep-Water formation rate during the last deglaciation, suggesting that variations in Nordic Seas deep-water circulation are forerunners to abrupt climate change; modelling of climate-carbon cycle uncertainties and their relation to the Paris Climate Agreement, arguing that a paradigm shift in impact evaluation is essential to support decision making and that estimates of mean patterns of

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change are insufficient; using a Gaussian process emulator of the likelihood surface with importance sampling to calibrate a stochastic computer simulation of moderate computational expense; showing that, for growing graphs with vertices embedded in space where the growth rules depend on a combination of preferential attachment to high degree vertices and spatial proximity, it was shown that as well as regimes where one of the two dominates as far as the behaviour of the degree distribution is concerned, there exists an intermediate regime which interpolates between the two, showing new, unexpected behaviour; it was shown that Markov chains of the type (X, J) , where $X \in \mathbb{N}_0$ is a nondecreasing position marginal, whose behaviour is modulated by the type $J \in \{1, \dots, \kappa\}$, have scaling limits which feature self-similarity properties.

1.3 Support for interdisciplinary research

Our research activity extends well beyond SoMaS, involving significant funding and collaboration with multiple groups and research centres within and outside the University. Maintenance of long-established partnerships and continual establishment of new ones allows us to generate critical mass and exploit funding opportunities. For example, within the University we work closely with the Faculty of Engineering (Solar Physics, Plasma Dynamics and Fluid Dynamics) and with Medicine, Biological Sciences and Computer Science (Mathematical Biology and Environmental Dynamics; Statistics). The Mathematical Physics group is part of an STFC funded consortium, jointly with Manchester and Lancaster Universities. These collaborations have attracted sustained external funding and are core to our plans to maintain and expand future research funding.

Examples of successful interdisciplinary research include:

- Juarez is the local Principal Investigator on an EU grant “In Silico Trial for Tuberculosis Vaccine Development” (STriTuVad consortium (2018–2023); EU contribution is €4.5M, with €465K attributed to Sheffield). The STriTuVad consortium is spearheading the use of computer simulations to supplement the information from physical clinical trials to decrease their length and duration, and hence reduce the cost of medicines. Juarez is developing a model that combines these two sources of information into an augmented Bayesian adaptive clinical trial.
- Rees has interdisciplinary collaborations with colleagues in Engineering and Molecular Biology and Biotechnology relating to the development of novel microbubble engineering technologies. These have been supported by a £2.2M EPSRC IB Catalyst grant (£929K to attributed to Sheffield) and a second EPSRC grant (£980K). Rees focuses on developing computational fluid dynamics models and solving inverse problems related to the engineering systems to inform experimentation and pilot plant operations. This research has potentially wide-reaching industrial applications, including bioethanol production (Monk is also involved in this part of the project) and plasma microreactors. Industrial partners include Perlemax, Suprafilt, Ensus, Vivergo Fuels, Vireol, AB Mauri and Heineken.
- Wilkinson secured funding from EPSRC on “Enhancing Machine Learning with Physical Constraints to Predict Microstructure Evolution” (£250K jointly with Clarke (Physics) and Cabral (Imperial College, Chemical Engineering)). He supervised the joint Sheffield postdoctoral researcher developing new methods for Bayesian optimisation for parameter estimation in spatio-temporal PDEs based on comparing physics model predictions with data.

Further examples of our successful interdisciplinary and collaborative research are summarised in Section 4.

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Building on the achievements of our research groups, we will continue to deliver key strands of our strategic plan by further developing research excellence through targeted recruitment in Algebra, Geometry and Topology and in Statistics. Specifically, in the short term we will recruit research leaders at chair level in both areas, with two linked lectureships in Statistics. We aim to invigorate and grow our current research groups by continuing to attract outstanding early career researchers with leadership potential, guaranteeing permanent positions for five-year fellowship holders.

Our planned recruitment in Statistics will support and increase our capacity for interdisciplinary research and impact generation (see below). To enhance this aspect of our research further, we will develop internal structures to coordinate our wide-ranging existing expertise in mathematical modelling, data analytics and simulation. This will position us to capitalise on emerging opportunities to deploy advanced mathematical and statistical methodology in new large-scale collaborations, particularly those addressing global challenges.

1.4 Impact strategy

We have an excellent track record of realising the economic, cultural, and societal impact of our research. We recognise that generating and developing impact requires long-term commitment, and we are committed to supporting our researchers in this. We benefit from expert professional services support to enable knowledge exchange and to support the development of impact, including commercialisation, impact development and business development. The team works closely with research group-contacts and individual researchers to identify, develop and fund new and existing impact and knowledge exchange opportunities. Other support for impact includes:

- Study leave (Section 2b) has proved valuable in developing impact from research. For example, Potts' earlier work had developed methods to predict brucellosis spread in elk populations and inform management decisions. These have been used to understand the effects of road building, forestry, climate change and human recreational activities on wildlife. Given these successes, Potts was granted study leave in 2018, during which he built on these techniques, leading to three publications, one of which builds on *Avgar et al.* (2016) to show how the technique can be used to parametrise models that can project forwards in time to make predictions of the effect of variables, which may be anthropogenically-driven, on the spatial structure of ecosystems. Such predictions are potentially very important for informing real-world management and conservation decisions. As a result, Potts secured an EPSRC project grant (£440K) to develop this research.
- We have supported Quegan's research over many years, by reducing teaching load and providing fee scholarships for a number of overseas PhD students (many from Beihang University), who made significant contributions to the research leading to his 2018 NERC Economic Impact Award.
- Fletcher is a lead developer of Chaste, a public C++ library for the numerical solution of mathematical models arising in physiology and biology. To promote new applications and development of Chaste, the University funded Fletcher to hold a Chaste workshop in Sheffield in August 2019, with 20 academic and commercial attendees (from Exeter, Bristol, Cardiff, Cambridge, Oxford, Warwick, Nottingham, and Microsoft Research).
- The University provides knowledge exchange support for researchers and academics who are engaging with external partners and in impact-led research. SoMaS researchers have secured funding from several of the available schemes, e.g. Heaton to attend the annual conference of the International Union for Quaternary Research in 2019 (£2K).

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- The Sheffield Elicitation Framework (SHELF) is a package of protocols, templates and guidance documents for conducting expert elicitation. The University supported the first SHELF user forum in 2019, a one-day workshop run by SoMaS with participants from academia and industry. Oakley also provided several training courses on SHELF methodology.

Going forward, enabling and developing impact from our research will continue to be a key priority. We will maintain support for established programmes that continue to generate new impact (e.g. IntCal, SHELF, BIOMASS). Drawing on our accumulated experience and the expertise of our knowledge exchange and impact support team, we will develop dedicated pipelines to nurture existing (e.g. Chaste, animal movement modelling) and develop new impact generation programmes. We understand the need to engage with stakeholders and the Statistical Services Unit provides an important mechanism for engagement with commercial and healthcare partners (see Section 3). We will further strengthen our research links with the SSU through statistical consultancy that involves development and deployment of advanced research methodologies.

1.5 Support for open research

We enthusiastically support a variety of approaches to open research and have engaged with the open access agenda for a long time. To ensure maximum discoverability of our research, we aim to put all outputs in White Rose Research Online, our shared repository with Leeds and York (green route). The 1,019 outputs deposited over this assessment period were downloaded over 32,000 times. This is our preferred route, ensuring equity in publishing opportunities regardless of available funding; we also publish outputs in fully OA journals or hybrid where required for funder compliance. In addition, staff regularly submit papers to preprint services such as arXiv to ensure timely dissemination of research findings. The principles of OA are made widely known to all staff, and open access guidance is an important part of the induction of early career researchers (ECRs).

We aim to make software openly available, as exemplified in our development of the public library Chaste, which: (i) exploits the breadth of expertise of a development team including mathematicians, computer scientists and biologists; and (ii) uses industrially derived techniques such as test-driven development to ensure code quality and reproducibility. SunPy, an open-source Python library for Solar Physics data analysis and visualisation has strong input from SoMaS. Stuart Mumford is a SunPy board member and lead developer who started work on SunPy while a PhD student in SoMaS and is now affiliated with SoMaS. The materials for SHELF (see above) are free to download, and the supporting software is open source. Oakley maintains an R package for SHELF, available on GitHub and on CRAN.

1.6 Support for research integrity

We are committed to the highest research standards in accordance with the University's Good Research and Innovation Practice policy. All PhD students undertake mandatory ethics and integrity training and prepare a data management plan before their 12-month confirmation review. All staff and students have access to online training in good research practices that reflects the national Concordat to Support Research Integrity. SoMaS engages with the Sheffield Reproducibility Network and the wider UK Reproducibility Network with the aim of sharing practices that enhance research quality, reproducibility, and reliability.

All research involving human participants, data or tissue follows the University's Ethics Review procedure. Our Ethics Administrator oversees ethics reviews and coordinates with the University

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Research Ethics Committee. They play a key role in raising awareness of, and providing guidance on, ethical issues and best practice in mathematical and statistical research. All research dissertations for our MSc and PhD programmes in Statistics require a specific ethics declaration. Our Statistics Clinic, which provides free statistical advice to researchers across the University, works to encourage and promote reproducible research more broadly. Future plans include:

- We have recently introduced a dedicated ethics reviewer panel within SoMaS to ensure that ethics reviewers have specific training and experience in ethics reviewing.
- We will provide more mathematics-oriented training examples for the compulsory PhD student training in research ethics (see Section 2).

2. People

2.1 Staffing and recruitment policy

Strategic recruitment sustains and develops excellence in our key research areas and provides support to facilitate research. We balance targeted recruitment to strengthen specific areas with wider calls to develop new areas and maintain breadth. We have approved plans for several new posts for the near future: to provide additional strategic leadership in pure mathematics we will appoint a chair; to strengthen research in Applied Statistics we will appoint a chair and then two lectureships (directed by the new Chair).

A key aspect of our research strategy to develop focused, dynamic groups of researchers, is to increase the numbers of ECRs will be our future research leaders, and, critically, to provide them with directed support and mentorship. Our 14 appointments during the assessment period were mostly ECRs (7 lecturers, 2 senior lecturers and 5 fellows). To help attract the best candidates and manage precarity, we now guarantee progression to an open-ended contract for all holders of long-term fellowships. All such fellows, and other ECR recruits, are guaranteed a funded studentship. Administrative and teaching loads are kept low, and an experienced mentor provides academic and pastoral support, enabling fellows to dedicate themselves fully to establishing their nascent research and developing outputs. We have used this approach to grow our Algebraic Geometry, Mathematical Biology and Mathematical Physics groups, with four fellows having transitioned to open-ended contacts. Whenever possible, we balance new growth with maintenance of critical mass in other groups (e.g. Probability, for which we have hired 3 new lecturers to replace one senior retiree).

In parallel to our direct research strategy, our recruitment is also guided by our postgraduate teaching programmes and commitment to deliver excellence in research-led teaching. Our appointments in algebraic geometry and mathematical physics allowed us to open a new MSc in Mathematical and Theoretical Physics in 2019 (run jointly with the Physics department); this programme contributes strongly to current and future research activity in these areas.

2.2 Staff development and support for our ECRs

Newly appointed academic staff have a mentor, who, in addition to the Head of School, is their primary point of contact. Our ECR lead plays a similar role for postdoctoral researchers and research fellows (who also have a mentor), supporting and nurturing ECRs in becoming research leaders. To allow newly appointed academic staff to develop their research, they have reduced teaching and administrative loads. They are particularly encouraged to supervise or co-supervise

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PhD students and to make use of departmental funding to stimulate research collaborations (see section 3), for which they receive priority.

All academic staff receive support for their research within their research groups and the School, e.g. by peer review of grant applications. The success of our system is reflected in the research grant awards to our current and recent ECRs, who had several successful grant applications throughout the assessment period: NERC (Potts, £144K), EPSRC (Berger, £331K; Brini, £1.2M; Campbell, £676K; Dolan, £98K; Hawkins, £99K; Potts, £350K; Shinder, £332K) and the Royal Society (Gielen, £423K; Manolache, £433K).

All staff have an annual review. Newly appointed ECRs have a specific review framework to support and develop them during their three years' probation. Established staff use the University's review scheme (Staff Review and Development Scheme (SRDS)), which provides a transparent and consistent framework. In both cases, reviews have retrospective and prospective elements, and are implemented so as to provide support and guidance to the reviewee. These processes also inform discussion of reward and recognition. The reviews cover all aspects of academic duties, including specifically research plans and progress towards publication, impact, and grants. Fixed-term ECRs receive dedicated support for CV and career planning, and, where appropriate, are offered opportunities for career development (e.g. teaching and research supervision).

As stated in the Institutional Statement (REF5a), the Academic Career Pathways (ACP) framework introduced in 2018, defines expectations and criteria for research, impact, teaching, leadership and professional standing which are appropriate both to career stage and to disciplinary norms. This framework thus provides a clear route for progression. To promote equity, progress against promotion criteria is considered for *all* academic staff each year.

Evidence of successful staff development is the outcomes achieved by our researchers. Our ECRs publish in leading journals such as *Inventiones mathematicae*, *Duke Mathematical Journal*, *American Journal of Mathematics*, *Physical Review Letters*, *eLife* and *Nature*. Berger, Dolan, Hawkins, Heaton and Potts have been promoted to Senior Lecturer, and Wilkinson from Senior Lecturer to Chair during the assessment period. Manolache and Brini were hired at the Senior Lecturer level during their Fellowships.

2.3 Support and reward for research and impact

We maintain a series of measures designed to protect research time, including a long-standing study leave policy in which members of staff are entitled to request a semester of leave after six semesters of standard duties. Study leave is used strategically to support research excellence and develop impact. Typically, 3–4 members of staff are on study leave at any one time; in total 35 semesters of study leave were granted during this assessment period. Study leave is often used for long-term research visits (including participation in major programmes such as those at the Isaac Newton Institute) and developing grant applications. For example, Potts and Blackwell used recent study leave to write successful grant proposals (Blackwell's Leverhulme fellowship on modelling wildlife movements and Potts' EPSRC award on multi-species aggregation models (Section 1b)). Externally-funded research time is accounted for in our workload allocation model. "Red-lining weeks" in January and July are cross-Faculty initiatives which provide protected time to enable staff to focus solely on research, and to develop new interdisciplinary collaborations and funding proposals. Wherever possible, research travel during the semester is accommodated by reallocation of duties. The distribution of administrative and teaching roles is reviewed on a yearly basis and is used to provide targeted skills and career development opportunities for staff (particularly ECRs).

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2.4 Support for postgraduate research students

2.4.1 Overview

Training PhD students is a vital part of our research activity; they are the next generation of mathematical scientists and we value the contribution our PGRs make to our intellectual environment and output. We currently have over 70 students in our PGR community. Doctoral awards in REF2014 averaged 12.75 per year; this has grown by 44% to an average of 18.3 over this assessment period.

Our vibrant PhD community conducts research across all our research groups. As a result of our substantial interdisciplinary research, our researchers also co-supervise PhD students registered in many other departments. Examples include Animal and Plant Sciences (Blackwell, Monk, Potts); Archaeology (Stillman); Biomedical Sciences (Fletcher, Monk); Computer Science (Quegan); Engineering (Blackwell, Oakley; Stillman); Geography (Heaton, von Fáy-Siebenbürgen); Clinical Medicine (Best, Fletcher, Monk, von Fáy-Siebenbürgen); Molecular Biology & Biotechnology (Monk); Physics & Astronomy (Juarez, Monk, Walters); Health and Related Research (Triantafyllopoulos). In addition, Katzman co-supervised a PhD student at Universidad Nacional, Medellin, Colombia and Fletcher at Oxford University. This extensive co-supervision network greatly enhances our research culture and the reach of our activity.

Oversight of PGR experience is through our PGR committee, chaired by the PGR Director. The committee comprises two academic members, and two student representatives. This ensures a balance between the supervisor and the student voice. PGR committee oversees all matters concerning PhD students, including recruitment, allocation of students to staff and allocation of funding to students. It is also responsible for oversight of students' professional development and monitors student progress.

2.4.2 Recruitment and funding

When recruiting PhD students, we aim to attract and recruit the most able candidates. Applications are considered by potential supervisors, to assess the academic suitability of candidates. The academic members of the PGR Committee consider cases for scholarship funding competitively; their primary criterion is academic strength, but balance between research areas, supervisory capacity and strategic areas for growth are also taken into account. PhD opportunities in the School are advertised nationally on our website.

We run a very active Undergraduate Research Experience Programme (with University-wide and SoMaS schemes) which encourages our undergraduate cohort to consider doctoral research and enables a focus on undergraduates from groups who are under-represented at PhD level.

In addition to our recurrent funding of PhD students through EPSRC and STFC (4 per year from EPSRC and 1 per year on average from STFC, plus CASE awards), we recruit excellent students via several University-level Centres for Doctoral Training (CDTs). These CDTs contribute strongly to our interdisciplinary research, particularly in Biological and Environmental Sciences.

We regularly attract students with funding from overseas (recent examples include funding from the Indonesian Ministry of Research, the Saudi embassy and from the Fundação para a Ciência e a Tecnologia, Portugal), contributing to the diversity in our PhD community. Between 2014 and 2018, we have increased the amount of PGR funding from overseas sources from 5.5 to 12 studentships.

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2.4.3 Development

In addition to their supervisory team, PhD students have an advisor who is well informed about their subject area and is involved in pastoral as well as formal support for the student. It is School policy that PGRs meet their PhD supervisor on a weekly basis (meetings are logged online).

On arrival, new PhD students participate in an induction event and undertake tailored training needs analysis with their supervisor and advisor to enable them to consider existing and desired skills and professional development. Development opportunities are offered at different organisational levels: SoMaS and the Faculty offer **research training** such as methods in mathematics and statistics training through MAGIC, APTS, and relevant MSc modules. For students in applied mathematics, we encourage scientific computing courses offered by the University's IT services. Interdisciplinary students can access training from outside SoMaS. All students are required to complete a Research Ethics & Integrity training module, delivered as part of a suite of research professionalisation modules by the Faculty.

Professional development and careers advice is available through the University *Think Ahead* programme, with a dedicated researcher development manager to advise and organise events. Provision includes academic and career mentoring, writing retreats, thesis bootcamps, student networks, such as disabled and ill students. Professional development opportunities also provide a network beyond the School.

Timely **progress** is supported by a number of milestones including a mid-year interim report (1–2 pages). 9 months after their start, students write a confirmation report (20 pages) and attend a review panel meeting shortly after this to discuss it and to get early experience of defending their research. This process is designed to be impartial, but the student's advisor is involved by way of support.

In year 2, students present a poster in the Faculty showcase, and at the end of the year they deliver a short talk to SoMaS PGR students and academic staff, and receive feedback. At the end of year 2, students write a progress report (5 pages), which is discussed with their supervisor and advisor. In year 3, there is a mid-year review report (1–2 pages) and an end of year three submission report designed to contain an outline of the thesis and the plan for completion. In year 4, quarterly reports are written by the supervisor, and responded to by the advisor and PGR Director.

To encourage networking and wider development, PGRs are encouraged to attend summer or winter schools, workshops and conferences and to give seminars either locally or in different institutions. All students are provided with a research training budget of £1.5K p.a. to support workshop/conference attendance and collaborative visits, and many make use of travel funds from external sources (e.g. academic societies). For example, PhD students have attended the British Universities Summer School in Elementary Particle Physics (STFC-funded), Tonale Winter School in Cosmology (Italy), STFC Introductory and Advanced/Topical Summer Schools and other specialist meetings. They are also strongly encouraged to attend various national or international conferences and workshops. Funding for these activities comes from STFC, EPSRC or SoMaS. While the attendance in summer/winter schools, conferences and workshops have educational benefit to the individual PGR, they can also lead to new collaborations. For example, Laura Paduraru collaborated with Peter Dunsby (University of Cape Town), resulting from discussions at the Iberian Cosmology Conference in 2015. Mark Yarrow published a paper in the *Electronic Journal of Linear Algebra*, after collaborative work starting during the Graduate Research Workshop in Combinatorics (a long-running programme which is organised by a group of universities in the US). Matthew Heathcoat has a contract funded by the World Resources Institute

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to detect tropical forest degradation using methods developed in his PhD. Hope Thackray visited the National Solar Observatory, Boulder (USA), for collaborative work.

Our PhD students also self-organise diverse activities, including a weekly PhD student seminar and reading activities for students across research groups. For example, the algebraic geometry and mathematical physics postgraduates organise reading groups where they work through a set of lecture notes on a particular topic. SoMaS PGRs organised and hosted the 2018 Research Students' Conference in Probability and Statistics.

2.5 Equality, diversity, and inclusion

We place equality, diversity, and inclusion (EDI) at the centre of our priorities, as emphasised by the first sentence of the SoMaS mission statement: "*The School of Mathematics and Statistics aspires to be an inclusive and welcoming environment for all who enjoy mathematics*". EDI matters are considered by our **Good Practice Group (GPG)**, which is chaired by the School Director for EDI, who also sits on the SoMaS Executive Committee. Other members of the GPG include the Head and Deputy Head of School. The aim of this group is to strengthen our focus on EDI and to mitigate risk of implicit bias, including in research matters. Input from the GPG has led to a number of activities and changes in policies to promote underrepresented groups and appreciation of EDI matters, including:

- the requirement for research groups to ensure appropriate gender balance when inviting seminar speakers;
- delivering annual implicit bias talks to all undergraduate students since 2018.
- running events to celebrate International Women in Mathematics Day;
- introduction of an annual undergraduate summer research scheme in 2019, organised and delivered by PGR students, that explicitly encourages applications from under-represented groups (in 2019, 4 out of 7 projects were awarded to female students and 1 to a non-binary student). This has recently received external funding from the Heilbronn Institute;
- creation of an 'Open CV' Project in 2019, with CVs of 12 academic staff at different grades to showcase different career paths;
- participation in the Faculty of Science Wellbeing Advocates programme, with three staff members associated with SoMaS taking on the role;
- contribution to the Faculty promotion framework in the area of metrics and gender bias.

It is University policy to offer staff flexible working arrangements, including working from home arrangements to accommodate, e.g. childcare and caring commitments. SoMaS also supports flexible working arrangements for staff nearing retirement by reducing their hours. Several staff take advantage of this flexibility.

Financial support for research is available to all staff members and applications from ECRs are highly encouraged and prioritised in order to increase their research visibility. Available support includes the Learned Society Fund, the Mathematics and Statistics Research Centre and the Research Stimulation Fund (see Section 3).

It is University policy that all chairs of all University interview panels must undertake training, which includes awareness of EDI issues. The University is a certificated Disability Confident Employer and SoMaS is committed to the key pledges of the Disability Confident scheme. To address

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gender imbalance, at the beginning of a recruitment process we form a search committee, which informs suitable applicants about job openings and encourages applications.

A key priority in the next few years will be to address the impact of the COVID-19 pandemic on our research-active staff. We recognise that the impact of the pandemic on research varies widely; to manage the resulting inequalities, we aim to use administrative/teaching load reduction and/or prioritised study leave to support those staff whose research capacity has been particularly affected by caring responsibilities and other factors during the pandemic.

Preparation for REF2021 We followed the institutional Code of Practice and members of our REF committee undertook EDI training provided by the University. All our eligible category A staff have been submitted. The number of outputs selected for each member of staff will not be used in any promotion or review process. An equality impact assessment found no systematic bias in our output selections.

3. Income, infrastructure and facilities

3.1 Income

During the assessment period our research income was **over £11M, a 111% increase** compared to £5.3M reported in REF2014. The majority of funding came from UKRI and the Royal Society (£7.0M), and the EU (£2.6M), as well as industry (£518K). We have significantly increased our EU funding since 2014 (when it was £199k).

Our strategy has been to diversify our funding portfolio, to grow grant income and to support our ECRs in writing grant applications. Notable successes include:

- £2.4M to Quegan, who has been supported by a wide variety of funders enabling his research leadership in the BIOMASS mission (including ESA, UK Space Agency, NERC, Leverhulme, East China Research Institute of Electronic Engineering).
- £1.8M to Bridgeland (including EPSRC, Royal Society, European Commission).
- Our current and recent ECRs have successfully competed for £4.1M of research grant funding from a variety of sources.

3.1.1 Strategies for successful research awards

To stimulate research grant applications, we adopt a dual strategy of raising awareness and providing tailored support.

Awareness: All staff receive regular targeted information about funding opportunities. The SRDS process (see Section 2b) provides a platform to discuss possible grant applications, and how best to meet research objectives. Where appropriate, staff access training and mentoring support for grantsmanship and proposal writing. Grant successes are celebrated in SoMaS and through a regular Faculty newsletter.

Support: Study leave is used to facilitate grant application writing (Blackwell and Potts have recently done so successfully, (see Section 2)). Dedicated professional research support enables applications, including financial aspects, coaching and coordination of mock fellowship interviews with academics who have served on relevant panels. To increase the competitiveness of proposals, we offer peer-support and review to refine proposals prior to submission, and mentoring

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for ECRs. Despite future uncertainties caused by the current pandemic and Brexit, we have laid the foundations to rapidly react to changes in the funding landscape.

3.2 Infrastructure and facilities

3.2.1 *The Statistical Services Unit (SSU)*

SSU is a specialist advisory unit providing comprehensive statistical consultancy services to clients within and outside the University. Based within SoMaS, it has a director (Sara Hilditch, who was awarded the Royal Statistical Society Chambers Medal for 2021) and 5 consultancy staff. Established clients include AstraZeneca, Unilever, Merseyrail and Chiltern Railways. Other work, in the medical/health field, includes supporting clinical research projects and clinical trials based in the Sheffield Hospitals Trust.

The expertise and contacts of the SSU provide an invaluable resource to foster and support academic and commercial research activity. Opportunities for methodological and applied research arise frequently through the SSU and our Statistics Clinic, but can be difficult to resource. To help exploit these, during the assessment period we have employed a postdoctoral research consultant to enable short to medium-term involvement in any academic research project that might benefit from statistical expertise. Projects have ranged from a week to several months, depending on the complexity of the problem, and are directed by a member of SoMaS research staff. These activities provide an important pipeline for knowledge exchange and future impact.

Exemplar projects include an investigation of the use of Bayesian methodologies to improve the quality of past climate reconstruction (directed by Heaton and colleagues at Cambridge University and CNRS/College de France), and a novel application of Bayesian Belief Networks to combine predictions from an ensemble of marine ecosystems and fisheries models, under different management scenarios, with statistical modelling of the economics of commercial fish prices (directed by Blackwell). This latter project brought together work carried out within the Marine Ecosystems Research Programme mainly by SoMaS, the Plymouth Marine Laboratory, the University of Strathclyde and the Centre for Fisheries and Aquaculture Science, and is expected to generate significant future impact.

Another function of our research consultancy is to run pilots to establish the viability of extended work. For example, using funding from the European Network for Business and Industrial Statistics to Stillman, we investigated the feasibility of using Bayesian methods for process optimisation. This has led to a PhD project supervised by Stillman, Blackwell and Majewski (Mechanical Engineering) using Bayesian errors-in-variables regression models to optimise 3D-printing processes.

3.2.2 *Computing*

Computing resource requirements for academics, PGRs, PDRAs and professional staff is reviewed annually with funds allocated to ensure all researchers have adequate computing resources for their needs. This includes the provision of high-power machines when required. In addition to local funds, specific computing requirements are funded by grant applications. Research data security and integrity are critical, and all research groups are allocated generous secure backed-up data storage capacity.

Some research groups, in particular Solar Physics, Plasma Dynamics and Fluid Dynamics, have significant computational needs, which led to the development of clusters of computing resources with high power, large storage, specialist software and excellent graphics capabilities. The University of Sheffield is a partner of the N8 Regional HPC Facilities partnership (with Durham,

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Lancaster, Leeds, Liverpool, Manchester, Newcastle, and York). It currently offers a shared Tier 2 HPC facility. Our IT Services provide licences for SAS, SPSS, Mathematica, Matlab and Maple. IT Services is committed to supporting research in all disciplines and strengthening relationships with researchers. It facilitates collaboration, supports research applications and helps departments with research costing management. It also maintains the ShARC and Bessemer high-performance computing clusters, of which Solar Physics, Fluid Dynamics and the Mathematical Physics cluster are heavy users. For example, ShARC is being used to acquire and pre-process large datasets from NASA's solar dynamics observatory database. Wherever possible, we are moving from local to University-managed virtual servers, to enhance robustness and security.

Two dedicated technical staff, together with the Director of Computing, advise on all aspects of IT. Biannual planning meetings plan for (i) incoming PhDs (all new PhDs are provided with a new machine according to a specification set out by their supervisor), and (ii) review staff machines, to maintain up-to-date IT provision for all staff.

3.2.3 Research visitors

We have hosted over 60 visitors during the assessment period, including a visit by the senior management team of the East China Research Institute for Electrical Engineering and the Vice-Dean of Beihang University (Quegan, 2019). We have dedicated office space for 8 short and long-term visitors. The Mathematics and Statistics Research Centre in SoMaS provides dedicated support and funding to support research visitors and small workshops. Applications for funding from the MSRC budget (£16k per year) are considered throughout the academic year; decisions are made by the Deputy Director of Research and two members of the Research Committee.

3.2.4 Learned Society (LSF) and Research Stimulation Funds (RSF)

We have dedicated investment to support external conference attendance or collaborative visits. The LSF supports small-scale activities for which funding is available without specific application, whereas more substantial activities are funded through RSF applications to the Research Committee. There is no limit (within reason) to the amount that can be requested for any activity, and most requests are funded in full.

3.2.5 The University Library

Sheffield has an extensive collection of learned journals and monographs in print and electronic form, in addition to many discipline-specific databases. The Library provides a mediated service for our White Rose Research Online (WRRO) repository (with Leeds and York) which removes burden from academic staff and ensures compliance with funder mandates.

4. Collaboration and contribution to the research base, economy and society

4.1 Research collaboration

Within Sheffield, we have an extensive and long-standing network of research collaborations, across science, engineering, and medicine. We have joint external funding and co-supervise PDRAs and PGRs. Jointly supervised researchers typically attend group meetings in more than one area, strengthening interdisciplinary links and enhancing their research training. These collaborations benefit from, and contribute strongly to, world-leading Sheffield facilities and infrastructure (for example, in biological imaging and clinical genetics). Many staff are affiliated with more than one research group, with particularly strong overlap between Mathematical Biology and

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Environmental Dynamics and Statistics, and growing interactions between Algebraic Geometry and Mathematical Physics. Cross-fertilisation between research groups is encouraged by regular well-attended “away days”.

We have many national and international collaborations, both long-established and arising more recently from new projects. The RSF (see Section 3) supports research visits, and this is regularly supplemented by external funding (e.g. current R. Soc. International Exchange grants to Bavula and Verth, Newton Fellowship to von Fáy-Siebenbürgen, Fulbright Scholarship to Best). During the assessment period, we have hosted more than 60 international visitors from the US, China, Brazil, Japan, Australia, South Africa and several European countries. More than 50% of our outputs have international co-authors.

Some exemplar collaborations include:

- Bridgeland’s link with the Universities of Edinburgh and Glasgow, leading to a £2.7M EPSRC programme grant on “Enhancing Representation Theory, Noncommutative Algebra and Geometry Through Moduli, Stability and Deformations”. This intra-disciplinary programme brings together a number of subfields of pure mathematics: algebraic geometry, representation theory and non-commutative algebra, by applying powerful cutting-edge techniques involving derived algebraic geometry and the theory of stability conditions. Project partners include the University of Toronto, MIT, University of Tokyo, University of Antwerp, University of Illinois Chicago, Kavli IPMU, University of Montpellier I, University of California, San Diego, and Tsinghua University.
- Quegan, with colleagues in the Biosciences, has long-standing central roles in the Leverhulme Centre for Climate Change Mitigation and the National Centre for Earth Observation, working on modelling of weathering dynamics and the carbon cycle.
- von Fáy-Siebenbürgen was coordinator of the €2.3M EU project PROGRESS (PRediction of Geospace Radiation Environment and Solar wind parameterS), which included researchers from Sheffield, the Finish Meteorological Institute, University of Warwick, Space Research Institute NSAU (Ukraine), Laboratoire de Physique et Chimie de l’Environnement et de Espace (France), Swedish Institute of Space Physics and the Helmholtz Centre for Geosciences (Germany). The project exploited spacecraft and ground-based data combined with state of art data assimilation methodologies to develop accurate and reliable forecasts of space weather hazards.
- Oakley and Wilkinson are currently working with Clayton (Computer Science) and Niederer (King’s College London) on cardiac modelling to predict which patients being treated for atrial fibrillation will go on to suffer from atrial tachycardia. The project is developing uncertainty quantification methods to allow for incorporation of patient measurements, and to enable clinical decisions that account for the inherent uncertainty present. This project is funded by EPSRC (2017–2021; £460K).
- Dolan, van de Bruck and Winstanley have, with the Universities of Manchester and Lancaster, established the “Consortium for Fundamental Physics”, which has been supported by STFC since 2011 (£976K, 2017–2021), involving cross-institution PhD collaboration and publication of jointly-authored papers.
- Campbell co-ordinated the Quantum Code Design & Architecture project with Terhal (TU Delft), Browne (University College London), Leverrier (INRIA, Paris) and Koenig (TU Munich). The project, funded through the EU and EPSRC co-fund Quant-ERA grant (2018–

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2021; 1.5M€), investigates the optimal design of quantum error correction codes to design efficient and reliable quantum computers and inform the design of hardware architectures.

4.2 Contributions to the economy and society

Much of our research in mathematical sciences can have significant economic, security and societal implications, often involving interdisciplinary collaboration. We encourage all staff to explore and develop such opportunities, and have a strong and sustained track record of generating impact from our research across the spectrum of our activity, including staff involvement in the Heilbronn Institute and GCHQ.

Our impact case studies provide compelling examples of the impact of our research. Further examples include:

- Wilkinson was funded by the Global Challenges Research Fund to develop and study networks of low-cost air pollution sensors. This links to a Google-funded project to build a network of sensors in Kampala, Uganda – a rapidly growing city with persistent dangerous levels of particulate pollution, exceeding the WHO's guideline annual mean limit by a factor of 10. The project aims to solve mathematical (a new machine learning paradigm for models involving space and time), engineering (detecting sensor malfunction, calibrating sensors remotely), and scientific (determining the sources of air pollution) problems. The project team is split between Sheffield and Kampala and works closely with the Kampala Capital City Authority and researchers from Google to develop possible interventions to improve air quality, model their potential impact, and measure their effectiveness.
- Best has an international network of collaborations with experimental biologists to study host-pathogen interactions and infectious disease dynamics. These include: (1) Dockrell (Edinburgh Medical School) and Marriott (Medical School, Sheffield), with whom he developed a model of the human immune response to *Staph. aureus* infections; (2) co-supervision of a PhD student with Brockhurst (Animal and Plant Sciences), where the student parameterised and confirmed their modelling work in an experimental bacteria-phage system; (3) long-term collaborations with biologists at Cambridge (Leggett), Exeter (Buckling) and UC Berkeley (Boots) using mathematical models to gain novel insights into results of experimental host-pathogen coevolution. Funding from a Leverhulme Early Career Fellowship (2013–2016) and a LMS Travel Grant (2019) has facilitated this work.
- Walters has worked with Cox (Oncology and Metabolism), Lin (Newcastle) and Michailidou (Cambridge) to develop a Bayesian approach that allows functional genomic information to inform breast cancer mutation effect size priors. Many forms of functional genomic information are incomplete, and they have developed a mixture model approach that incorporates differential shrinkage to allow for this incompleteness. They have also shown how estimates of the number of yet-undiscovered causal mutations can be used to inform this effect size prior and how uncertainty in this estimate can be incorporated.
- Fletcher received funding from the BBSRC to study the integration of growth and patterning during animal development with Strutt (Biomedical Science) and Richmond (Computer Science). The team combines experiments with mathematical and computational modelling to explore how cells in the developing fruit fly wing communicate to build a sense of direction, how this sense of direction orients growth, and how cell-level information provides feedback to the system to halt growth. This work utilises Chaste (see Section 1.4), an open-source software library for cell-based and multiscale modelling developed by Fletcher and colleagues, including Oxford (Byrne, Gavaghan, Maini), Nottingham (Mirams), and

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Melbourne (Osborne). Fletcher has led several Chaste training workshops for mathematical biologists, including in Sheffield, Dundee, and Norwich, with financial support from the EPSRC-funded POEMS network, the BBSRC-funded Multiscale Biology Network, and Sheffield knowledge exchange funds.

- Blackwell and Heaton are key long-standing members of the international working group on radiocarbon calibration curves (IntCal), which provides the international standards for determining age estimates from ^{14}C measurements. Sheffield plays a lead role in the development of statistical methods underpinning this programme, which has significant impact on environmental policy and the heritage sector. Heaton provided statistical leadership in the recently (2020) released major update of the calibration curves (IntCal20).

4.3 Contribution to sustainability of the discipline

We continually support the research base of our discipline in several ways. Our workload allocation model supports our researchers to devote time to community activity such as editorial work, external examining or chairing external committees. Editorial board memberships include: Bridgeland (*J. European Math. Soc.*); Fletcher (*Biophys. J., Math. Biosci. & Engineering*); Jain (*Astrophys. J. Lett.; Astrophys. J.*); Jordan (*J. Appl. Prob.; Adv. Appl. Prob.*); Oakley (*J. Uncertainty Quantification*); Willerton (*Compositionality; J. Homotopy & Related Struct.*); Winstanley (*Classical and Quantum Gravity*).

Research staff regularly contribute to national and international scientific boards, including:

- Quegan has been an invited member of numerous significant national and international advisory committees, including (since 2013): chairman of the ESA BIOMASS Mission Assessment Group; member of the Kyoto and Carbon Advisory Group; member of the Terrestrial Observations Panel on Climate (an advisory panel to the UN Framework Convention on Climate Change); leader of the Group on Earth Observation Forest Carbon Tracking task. He also represented NERC at the House of Commons Science and Technology Select Committee's Inquiry into UK and European Space Agencies (June 2013).
- Oakley was a member of the expert review committee for the EPSRC/Knowledge Transfer Network review of knowledge exchange in the mathematical sciences (published 2018) and is also on the scientific advisory panel for the Newton Gateway to Mathematics (the impact initiative of the Isaac Newton Institute).
- Wilkinson has joined the Data Evaluation and Learning for Viral Epidemics (DELVE) group, a multi-disciplinary group convened by the Royal Society during the COVID-19 outbreak to provide input to SAGE, the scientific advisory group for emergencies. Wilkinson worked on the evidence base for the use of masks and co-authored a paper on the effect reopening of schools on the development of the pandemic.
- Winstanley has been elected to the Nominating Committee in the International Society for General Relativity and Gravitation (2016–2022).
- Hawkins was on the IOP Science and Innovation Committee (2014–2018).

We have organised a number of workshops and conferences in our research areas, including:

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- **Probability:** Jordan and Freeman organised the UK Easter Probability meeting in 2018, funded by grants from EPSRC, the LMS, the Heilbronn Institute and the Applied Probability Trust.
- **Mathematical Biology:** Best co-organised in 2018 a mini-symposium in Lisbon on “Current directions in host-parasite coevolution”; Potts was involved in organising the Annual Meeting of the Movement Ecology Special Interest Group within the British Ecological Society (2017; London) and a joint annual meeting of the Movement Ecology and Quantitative Ecology Special Interest Group within the British Ecological Society (2019, Sheffield); Fletcher has organised 11 conferences and workshops since 2014, including a Royal Society Discussion Meeting on “Contemporary Morphogenesis” (2019, London) and a Mathematical Research Institute (MATRIX) Workshop on “Virtual tissues: progress and challenges in multicellular systems biology” (2018, Melbourne); Monk organised a mini-symposium on “Process Biology”, Society for Experimental Biology Annual Meeting (2015, Prague) and was organiser and tutor for two EMBO Practical Courses on “Multi-level Modelling of Morphogenesis” (2015, 2017); Quegan was co-organiser of the International Workshop on Space-Based Measurement of Forest Properties for Carbon Cycle Research (2017, International Space Science Institute, Bern).
- **Number Theory:** Şengün was involved in organising the conference “Arithmetic groups and 3-Manifolds” (2020, University of Hagen), and meetings on “Arithmetic and Low Dimensional Hyperbolic Spaces” (2016, University of Galatasaray), “Cremona 60th: Explicit Methods in Number Theory” (2016, Warwick) and “Frontiers in Serre’s Conjecture: Torsion and Low Weights” (2015, University of Luxembourg); Berger organised the Second international conference on Galois representations and automorphic forms (2016, Będlewo), and a meeting on “p-adic modular forms and Galois representations” (2019, Sheffield).
- **Solar Physics and Plasma Dynamics:** von Fáy-Siebenbürgen was a member of the scientific organising committee for more than 10 conferences/meetings, including the AGU Fall Meeting (2019, San Francisco), ST20, Asia Oceania Geosciences Society (2019, Singapore) and the India-UK Seminar on “Plasma Processes in the Solar and Space Plasma at Diverse Spatio-Temporal Scales: Upcoming Challenges in the Science and Instrumentation” (2014, Nainital); Jain was the convenor of the session ST06 at AOGS Meeting in (2019, Singapore); Ballai was the main organiser of the 5th UK-Ukraine-Spain meeting on Solar Physics and Space Science (2019, Ukraine).
- **Algebraic Geometry and Topology:** Bridgeland and Shinder organised a two-day workshop on “Homological algebraic geometry” (2016, Sheffield); Bridgeland and Meinhardt organised a workshop on Stability conditions, Donaldson-Thomas invariants and cluster varieties (2017, Sheffield) and organised a workshop “Stability conditions” (2015, Warwick); Shinder has co-organised a number of workshops since 2014, including the workshop on “Derived categories, motives and Zeta-functions” (2014, Edinburgh), the Sheffield GLEN workshop in 2018 and the workshop on “Constructions and Obstructions in Birational Geometry” at the International Centre for Mathematical Sciences, Edinburgh (2018). Willerton was on the programme committee of the international Applied Category Theory conference (2019, Oxford); Whitehouse was involved in several Woman in Topology events: she was a team leader at the meeting in Banff in 2013 and 2016 and organiser of the third Women in Topology meeting in Bonn in 2019.

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- **Mathematical Physics:** Gielen was a co-organiser of “Quantum Gravity 2020”, a major international conference in this research area. Gielen and van de Bruck organised the UK Cosmology meeting in 2020.

SoMaS has hosted the **Applied Probability Trust**, an independent not-for-profit foundation for study and research in the mathematical sciences, since its establishment in 1964. The Trust publishes the *Journal of Applied Probability* and *Advances in Applied Probability* (with CUP).

Visiting professorships: Ballai held a visiting professorship in 2017 (Universitat des les Illes Balear); von Fáy-Siebenbürgen had several visiting research professorships, including at the Chinese Academy of Science Hefei (2019); National Astronomical Observatory of China, Beijing (2016 and 2017); and Debrecen Heliographic Observatory, Hungarian Academy of Science Distinguished Visiting Professorship (2014). Katzman was visiting professor at the University of Illinois, Chicago.

Fellowships: The research contributions of our staff have been recognised with a number of fellowships, including Blackwell and Heaton (Leverhulme Research Fellowships), Brini and Campbell (EPSRC Early Career Fellowships), Manolache (Dorothy Hodgkin Fellowship), Gielen (Royal Society University Research Fellowship).

In summary, we are proud of our important contributions across the spectrum of mathematical science, demonstrated by the quality of our research outputs and resulting impact. Embedded in a vibrant research community within Sheffield and supported financially by a wide range of external sources, we are well placed to build on our successes to make further significant contributions to mathematics and statistics in the next decade.